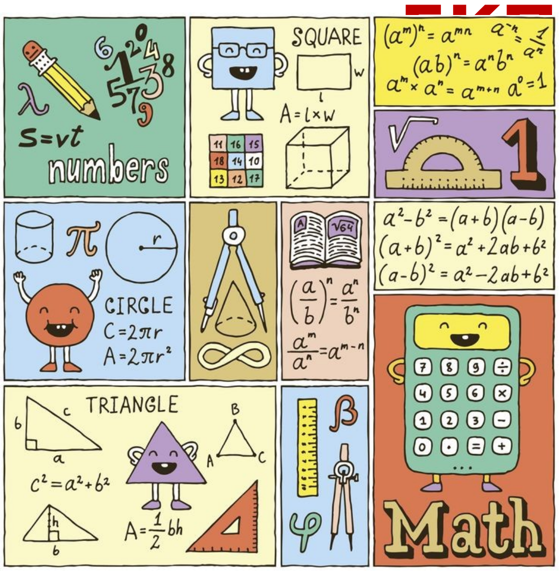
ΣΥΛΛΟΓΉ ΜΑΘΗΜΑΤΙΚΏΝ ΕΙΚΌΝΩΝ ΓΙΑ ΤΗΝ ΔΕΥΤΕΡΟΒΆΘΜΙΑ ΕΚΠΑΙΔΕΥΣΗ



ΕΠΙΜΈΛΕΙΑ ΣΥΛΛΟΓΉΣ: ΓΙΆΝΝΗΣ Π. ΠΛΑΤΑΡΟΣ ΣΎΜΒΟΥΛΟΣ ΕΚΠΑΊΔΕΥΣΗΣ ΜΑΘΗΜΑΤΙΚΏΝ.

Μαθηματικές Εικόνες από pinterest

Πρόκειται για μια Συλλογή στατικών εικόνων από το pinterest εδώ: https://gr.pinterest.com/

1. Πρόκειται για μια συλλογή εικόνων παγκόσμια.
2. Αφορά ταξινομήσεις όλων των ειδών, όλων των ενδιαφερόντων με μια ποικιλία πραγματικά απίστευτη και ανανεούμενη.
3. Όποιος φτιάξει λογαριασμό στην πλατφόρμα αυτή μπορεί να φτιάξει τις δικές του υποκατηγορίες και συλλογές .
4. Οι παρατιθέμενες εικόνες έχουν συλλεχθεί με τα εξής κριτήρια:

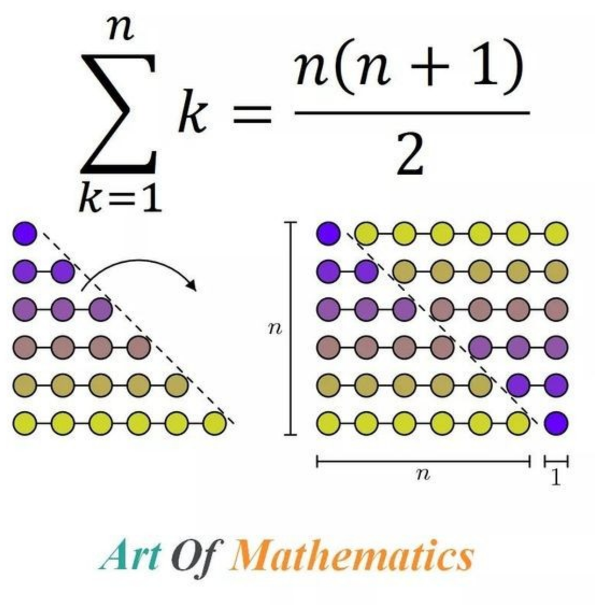
- Μαθηματικά Δευτεροβάθμιας.
- Αναγκαστικά ακίνητες εικόνες (αποκλείονται gif και βιντεάκια.)
- Για κάθε παρατιθέμενη εικόνα υπάρχουν εκατοντάδες παρόμοιες .
- Να έχουν μια σαφή προστιθέμενη αξία σε επίπεδο παραστατικότητας και διαφορετικής αναπαράστασης κλασικών εννοιών.
- Για χρήση του εκπαιδευτικού είτε των μαθητών.

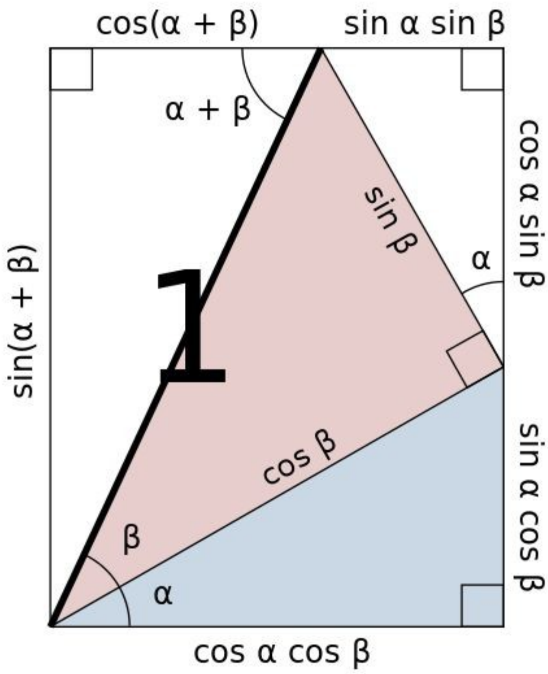
5. Οι εικόνες είναι αποθηκευμένες με πλήρη ανάλυση από την πηγή, αλλά έχουν μικρύνει για λόγους χώρου.
6. Οι περισσότερες ανήκουν στην κατηγορία «αποδείξεις δίχως λόγια»
7. Υπάρχουν συλλογές και ταξινομήσεις για οτιδήποτε αφορά ανθρώπινη δραστηριότητα είτε ενδιαφέρον.
8. Μια κατηγορία που προτείνουμε οπωσδήποτε είναι οι οφθαλμαπάτες (Illusions)
9. Για να χρησιμοποιηθεί αποδοτικά η συλλογή, πρέπει να είναι σε έκδοση .docx και όχι .pdf Θα υπάρχει στην επιλογή file της ανάρτησης
10. Η πλατφόρμα λειτουργεί πολλά χρόνια είναι πολύ δημοφιλής στην Αλλοδαπή και υπάρχει τεράστια ποικιλία στις συλλογές.

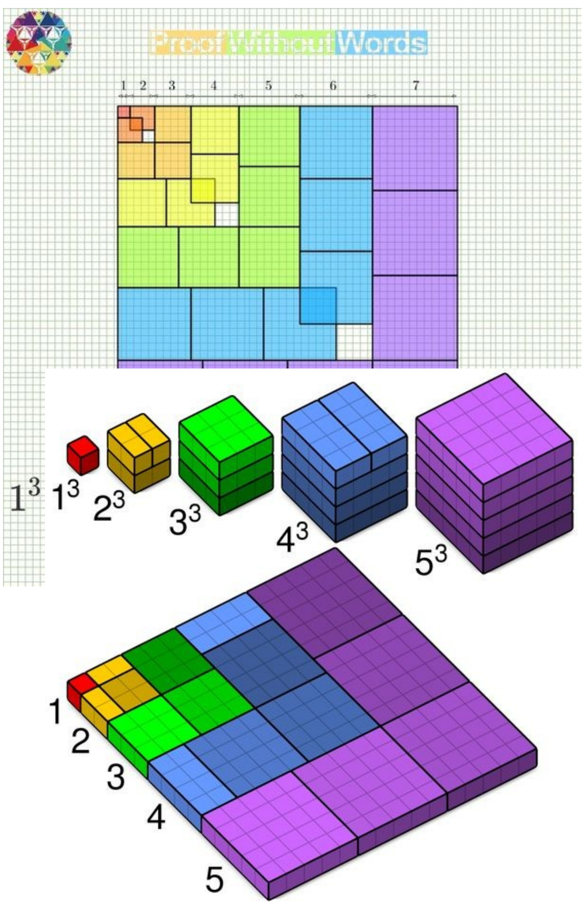
11. Προτείνουμε και άλλες συλλογές πλην του pinterest στην δικτυογραφία.
12. Το βιβλίο του Ρότζερ Νίλσεν [υπάρχει εδώ σε .pdf]() (το πρώτο από τα τρία . [Υπάρχει και εδώ]()

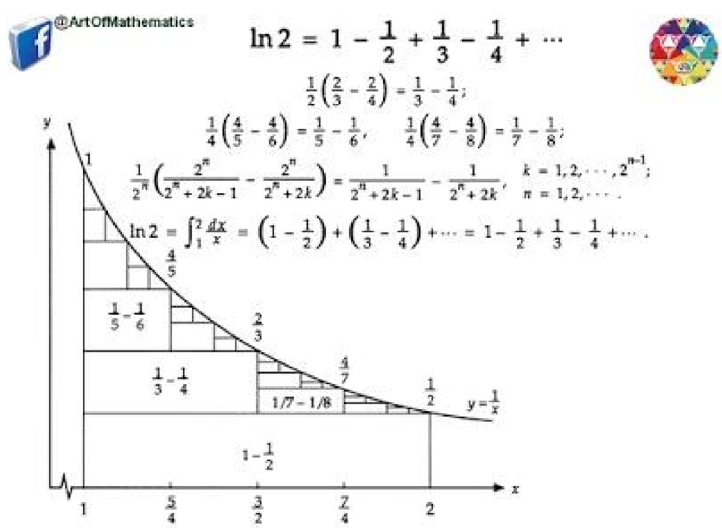
Γιάννης Π. Πλατάρος

Μαθηματικός





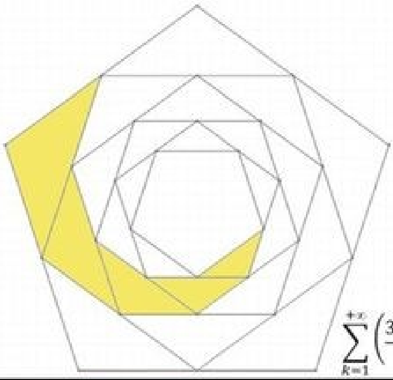




Geometrical aspect **Algebraic aspect**

$$\sum_{k=1}^{+\infty}\frac{1}{4^k}=\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+\cdots=\frac{1}{3}$$

$$\sum_{k=1}^{+\infty}\frac{1}{2^k}=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots=1$$



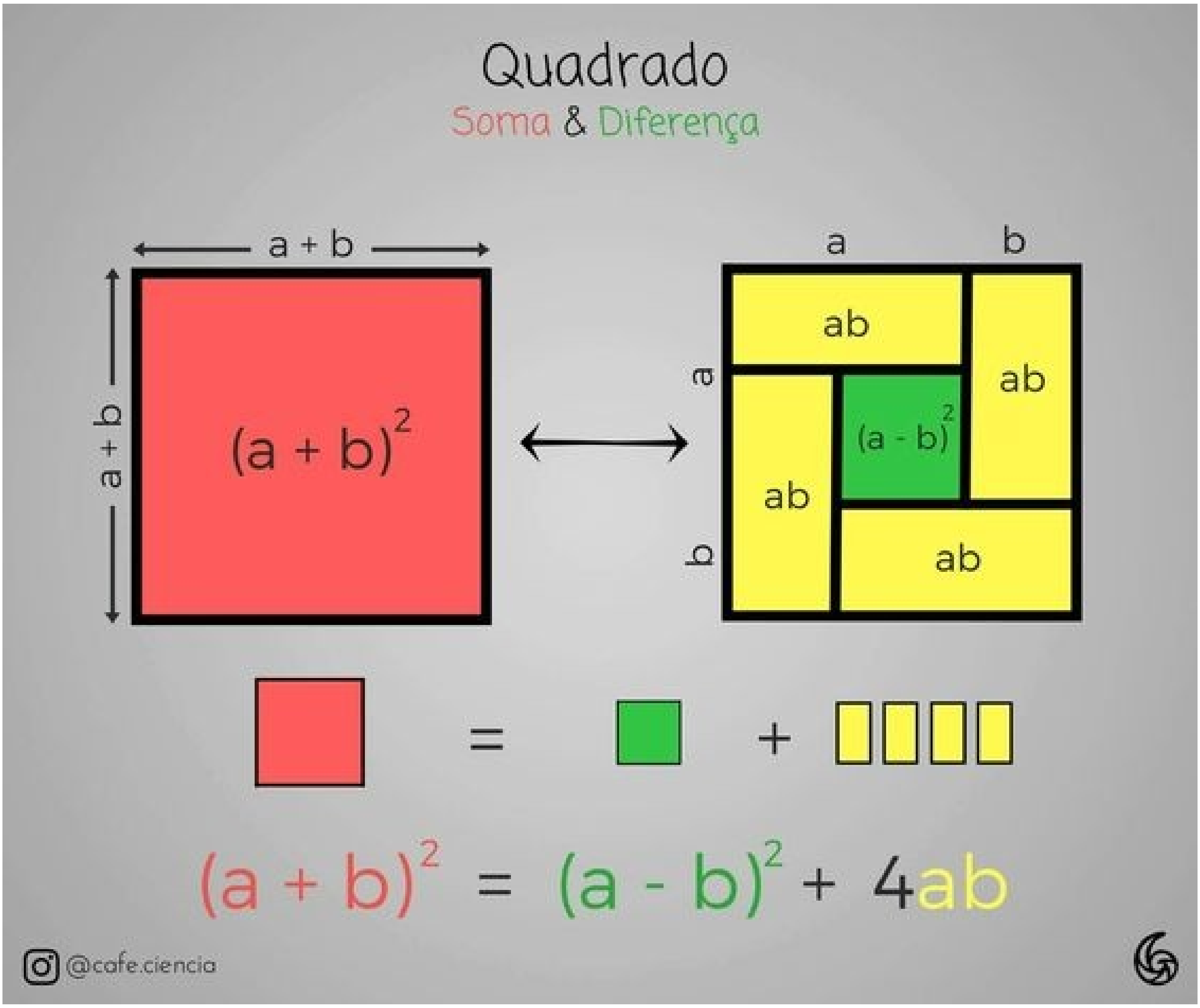
$$\sum_{k=1}^{+\infty}\left(\frac{3+\sqrt{5}}{8}\right)^k=\left(\frac{3+\sqrt{5}}{8}\right)+\left(\frac{3+\sqrt{5}}{8}\right)^2+\left(\frac{3+\sqrt{5}}{8}\right)^3+\cdots=\frac{1}{5-2\sqrt{5}}$$

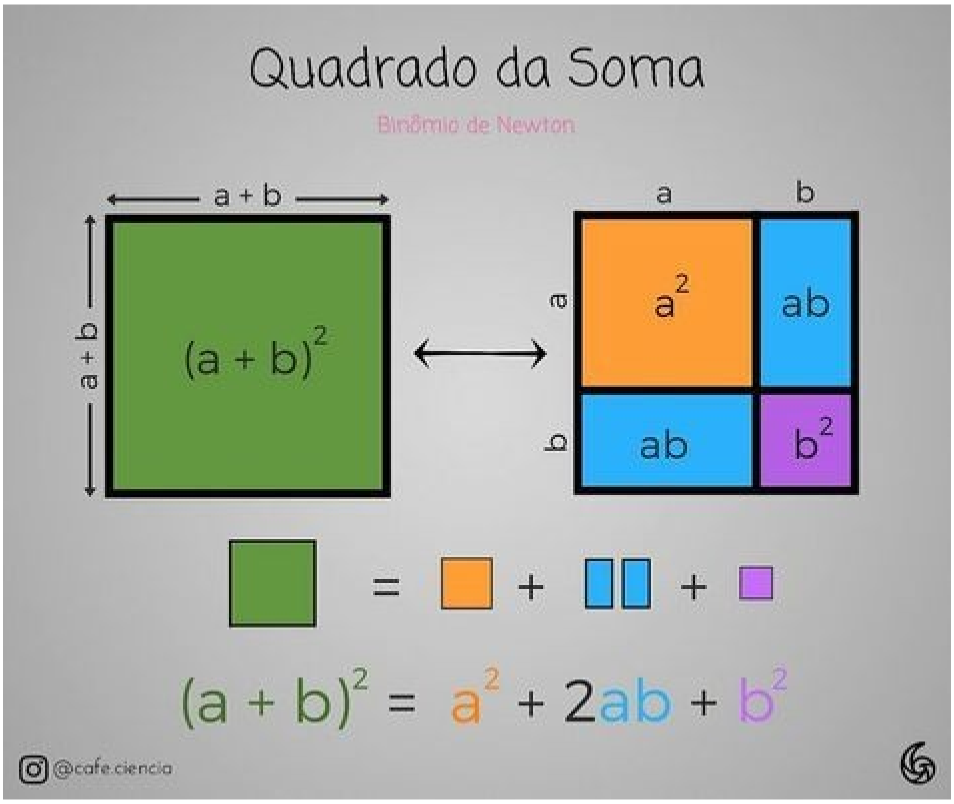
After realizing the meaning above

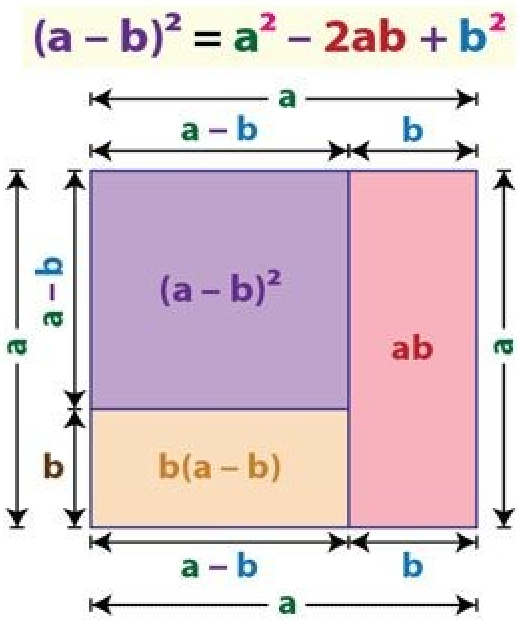
Find the algebraic aspect of this pattern I will post the best solution with your name and your photo.

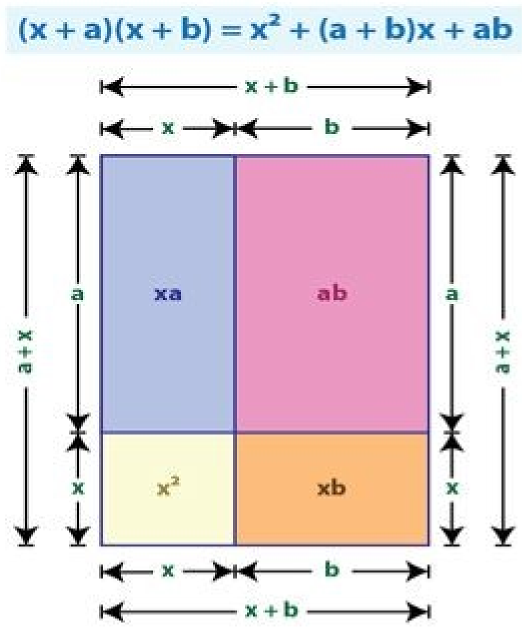
?

Art Of Mathematics


Quadrado
Soma & Diferença
a + b
a + b
$(a + b)^2$
a
b
a
b
ab
ab
ab
ab
$(a - b)^2$
=
+
$(a + b)^2 = (a - b)^2 + 4ab$
@cafe.ciencia

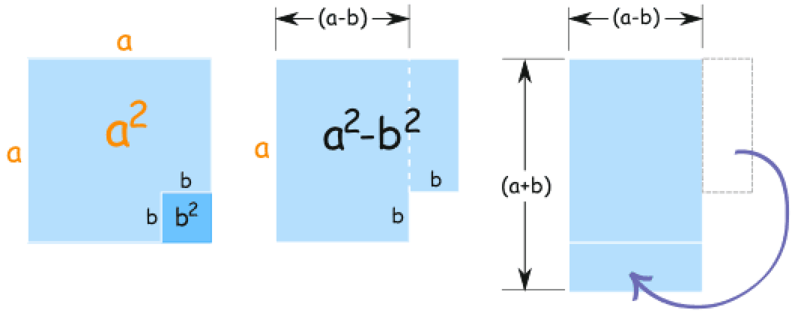


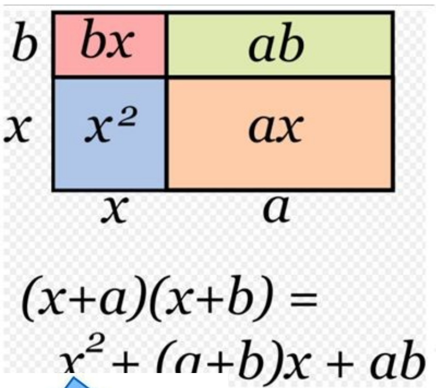


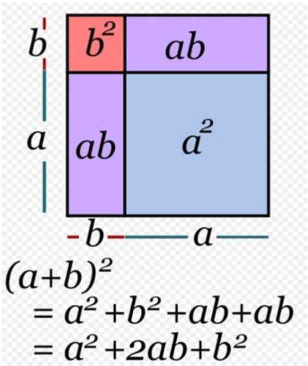


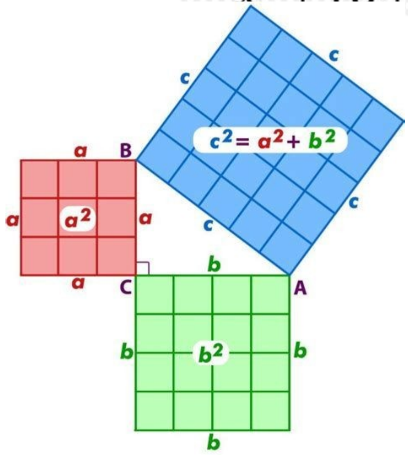
$$\sqrt{2} + \sqrt{3} \overset{?}{=} \pi$$

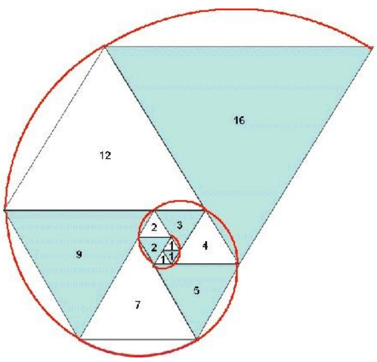
$$\begin{array}{rl} & \sqrt{2} = 1,41\ldots \\ + & \sqrt{3} = 1,73\ldots \\ \hline & \pi = 3,14\ldots \end{array}$$

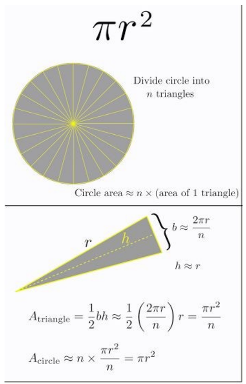

a
a
a^2
b
b
b^2
(a-b)
a
a^2-b^2
b
b
(a-b)
(a+b)

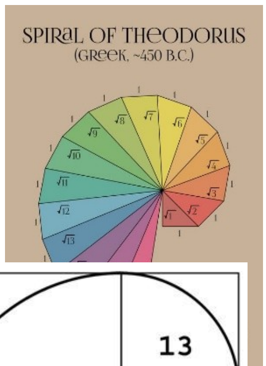

b
bx
ab
x
x^2
ax
x
a
$(x+a)(x+b) =$
$x^2 + (a+b)x + ab$

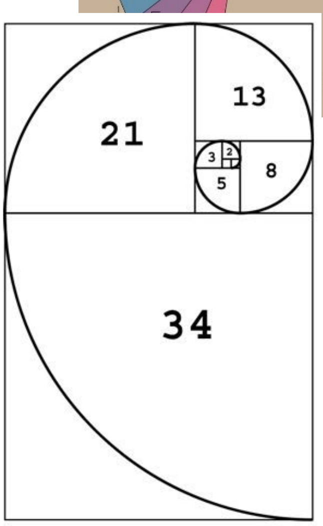

b
b^2
ab
a
ab
a^2
b
a
$(a+b)^2$
$= a^2+b^2+ab+ab$
$= a^2+2ab+b^2$

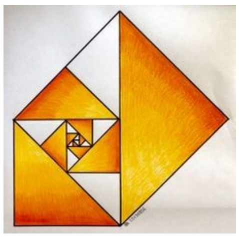

$c^2 = a^2 + b^2$
a^2
b^2
A
B
C

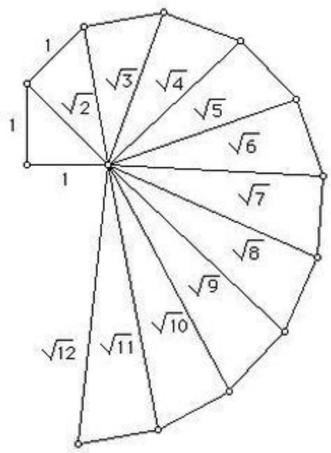

16
12
9
7
5
4
3
2

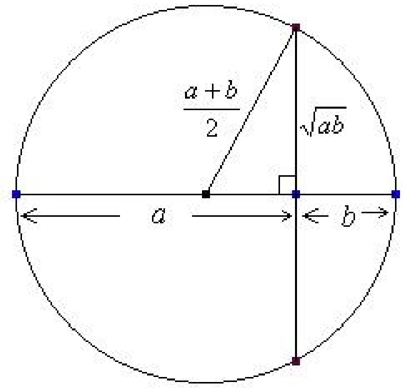

πr^2
Divide circle into
n triangles
Circle area ≈ n × (area of 1 triangle)
$b \approx \frac{2\pi r}{n}$
$h \approx r$
$A_{\text{triangle}} = \frac{1}{2}bh \approx \frac{1}{2}\left(\frac{2\pi r}{n}\right)r = \frac{\pi r^2}{n}$
$A_{\text{circle}} \approx n \times \frac{\pi r^2}{n} = \pi r^2$

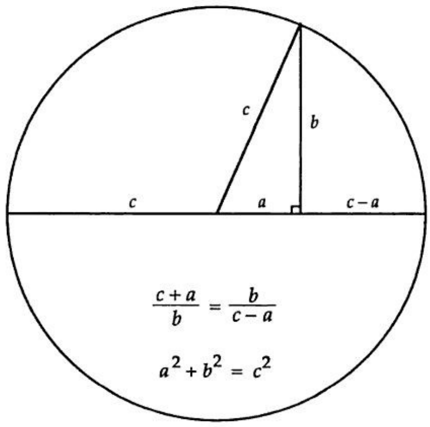

SPIRAL OF THEODORUS
(GREEK, ~450 B.C.)

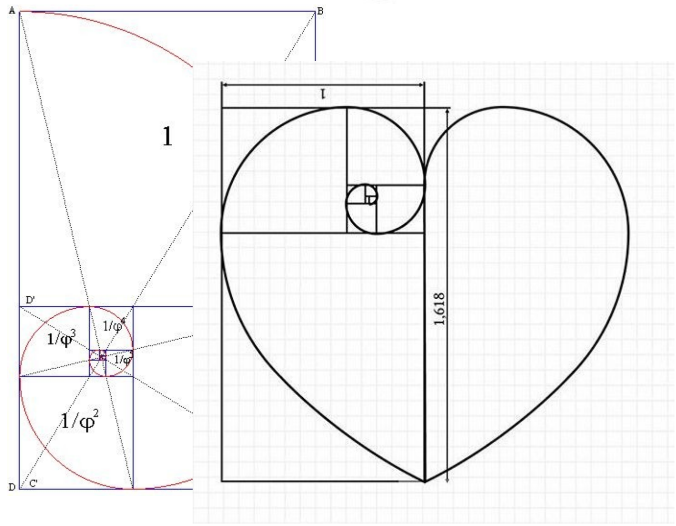

21
13
3
2
5
8
34

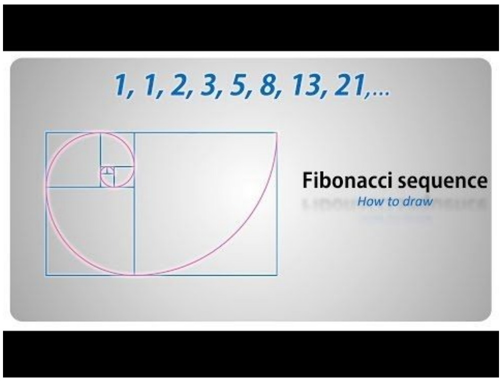


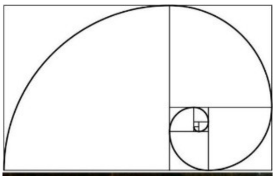

1
1
1
$\sqrt{2}$
$\sqrt{3}$
$\sqrt{4}$
$\sqrt{5}$
$\sqrt{6}$
$\sqrt{7}$
$\sqrt{8}$
$\sqrt{9}$
$\sqrt{10}$
$\sqrt{11}$
$\sqrt{12}$

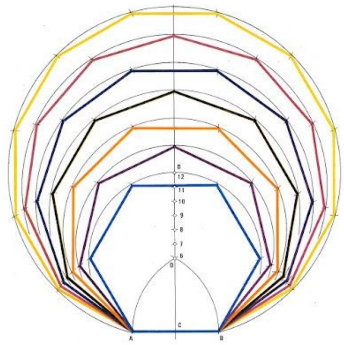

$\frac{a+b}{2}$
$\sqrt{ab}$
a
b

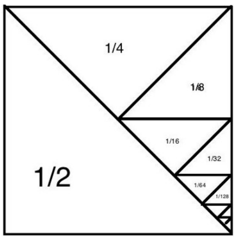

c
b
c
a
c − a
$\frac{c+a}{b} = \frac{b}{c-a}$
$a^2 + b^2 = c^2$

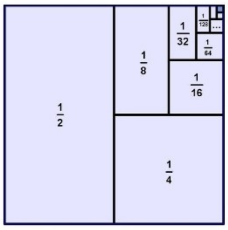

A
B
1
D'
D
C'
$1/\varphi^3$
$1/\varphi^4$
$1/\varphi^2$
I
1,618

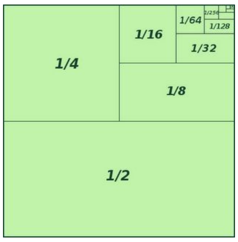


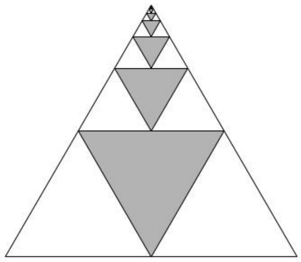




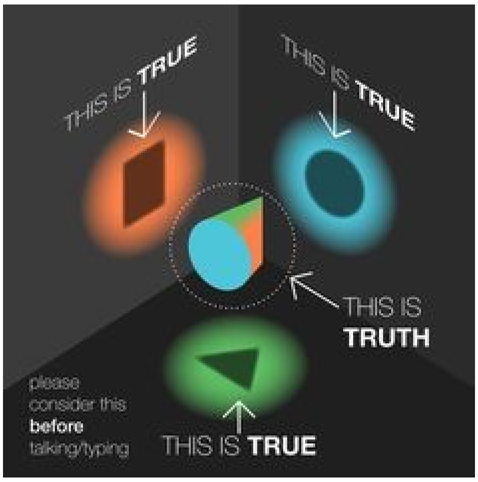


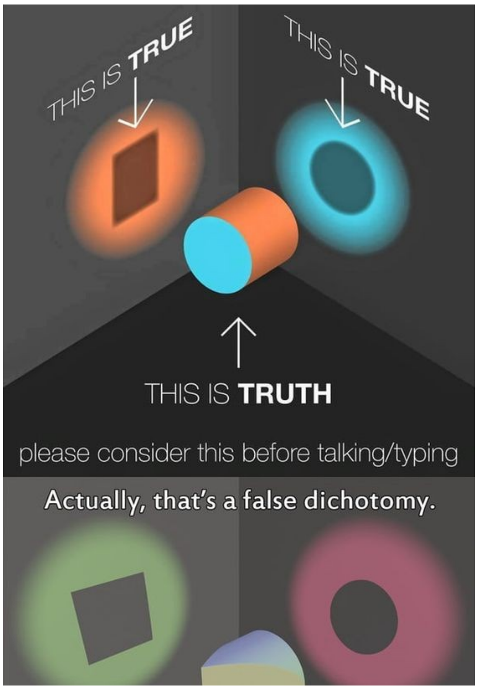


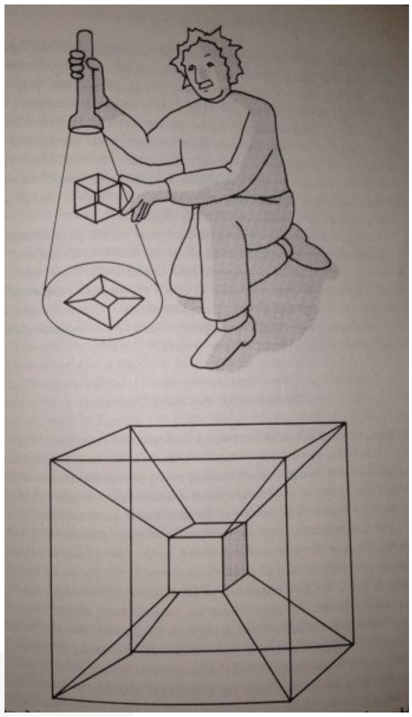


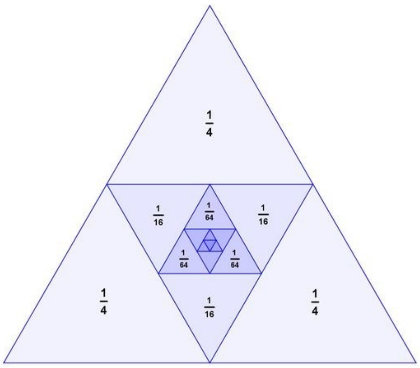


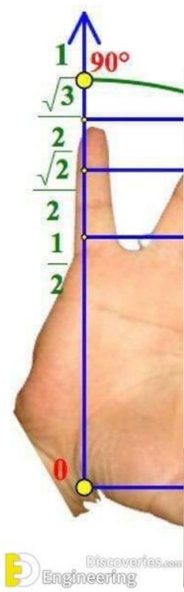
$$\frac{1}{3} = \sum_{i=1}^{\infty} \frac{1}{4^i} = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \frac{1}{4^5} \cdots$$

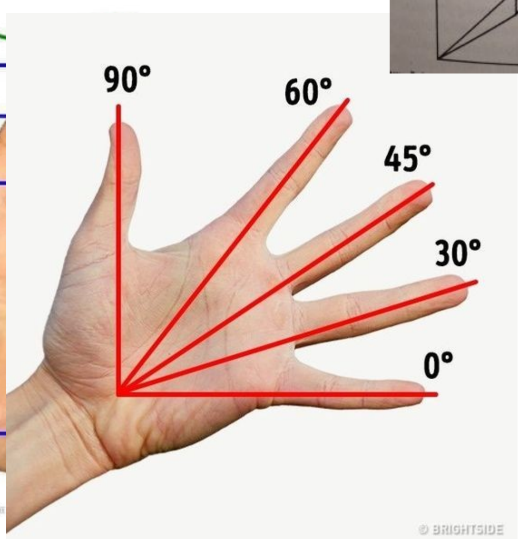


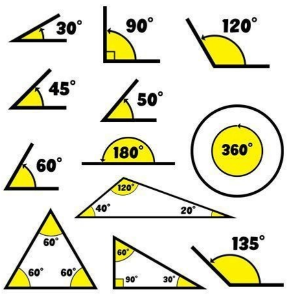


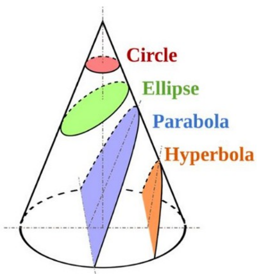


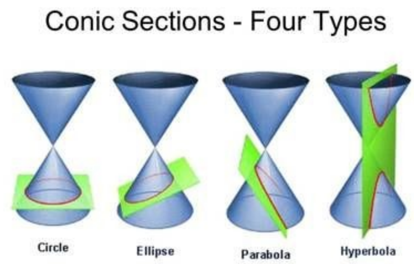


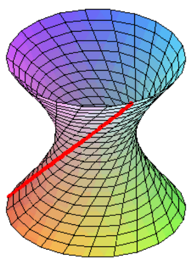


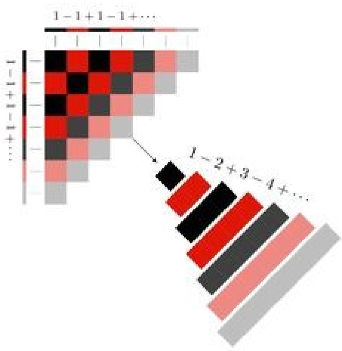


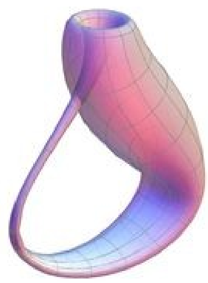

30°
90°
120°
45°
50°
60°
180°
360°
120°
40°
20°
60°
60°
60°
60°
90°
30°
135°


Circle
Ellipse
Parabola
Hyperbola

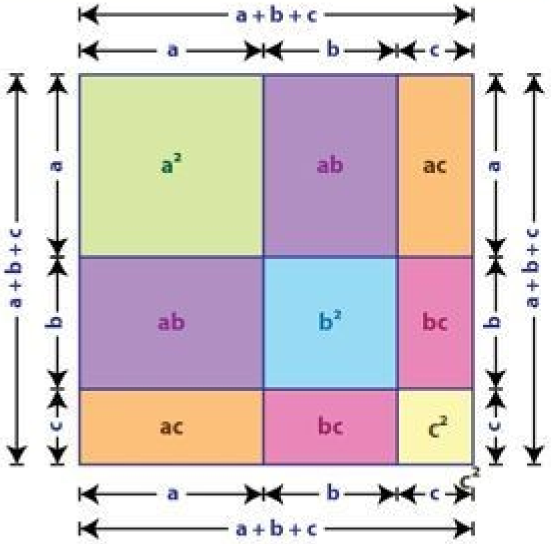

Conic Sections - Four Types
Circle
Ellipse
Parabola
Hyperbola

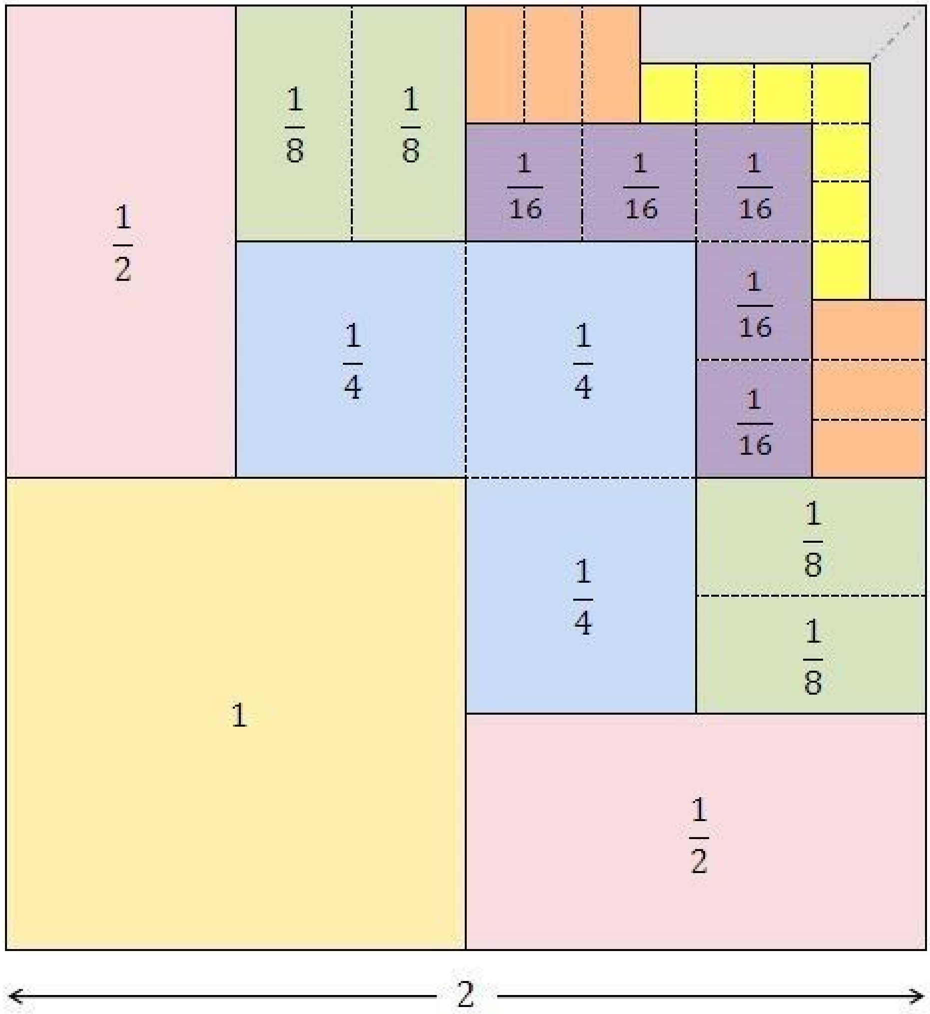


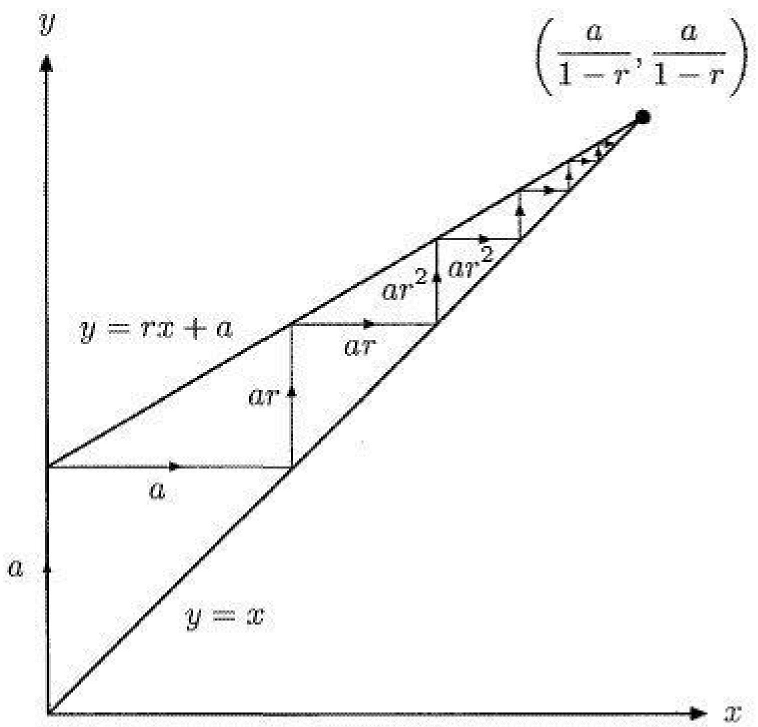

1 − 1 + 1 − 1 + ⋯
1 − 2 + 3 − 4 + ⋯



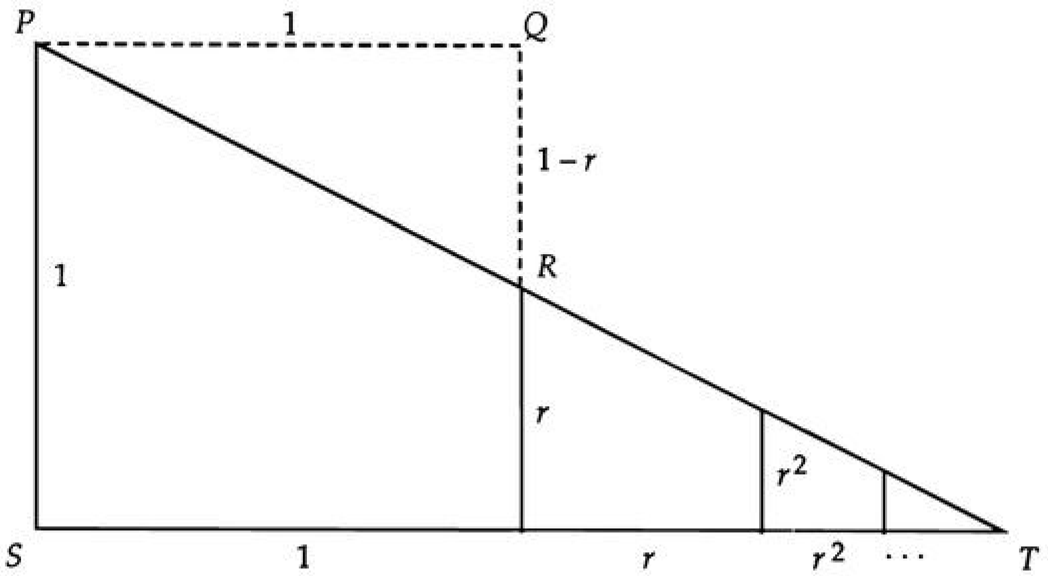
$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$





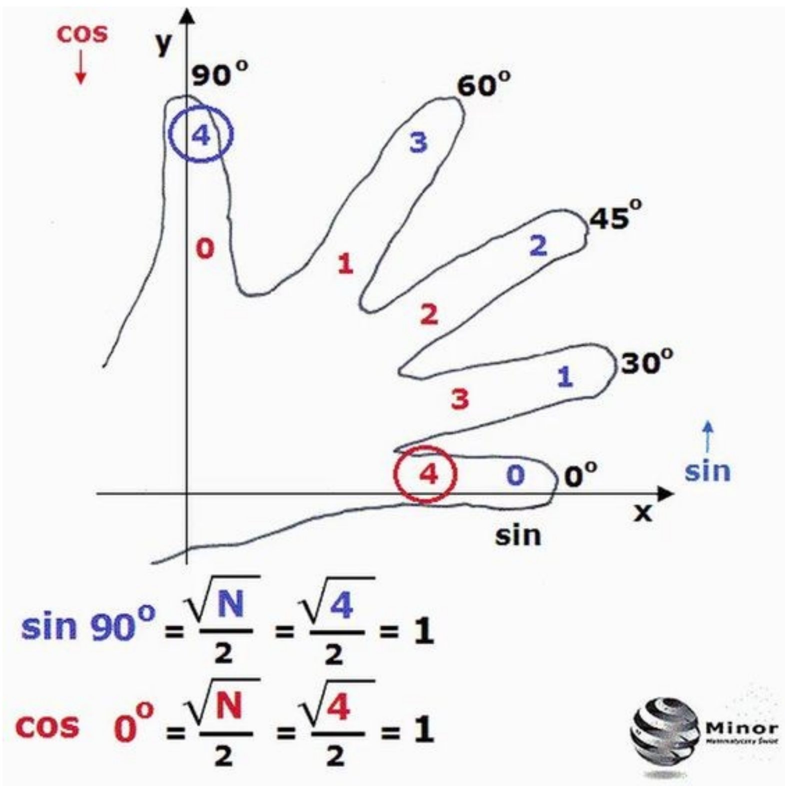


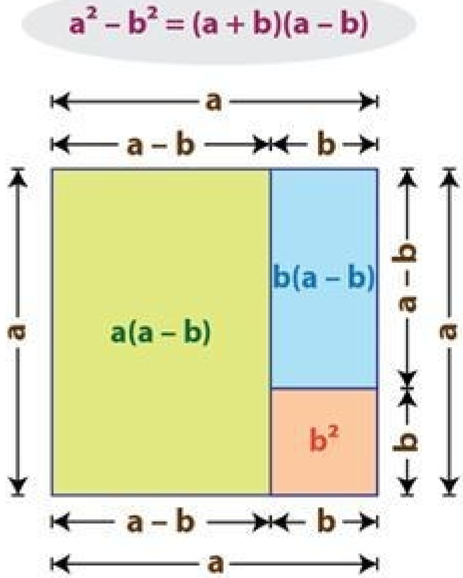
$$a + ar + ar^2 + \cdots = \tfrac{a}{1-r}, \quad 0 < r < 1.$$

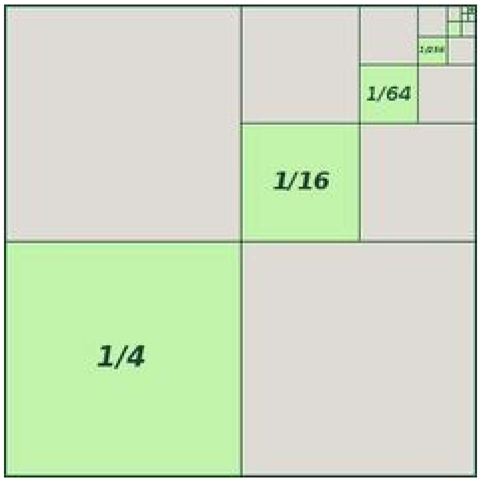


$$\Delta PQR \approx \Delta TSP$$

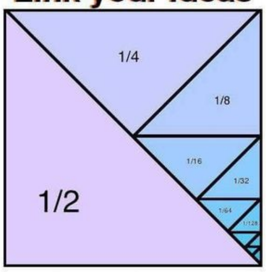
$$\therefore 1 + r + r^2 + \ldots = \frac{1}{1-r}.$$







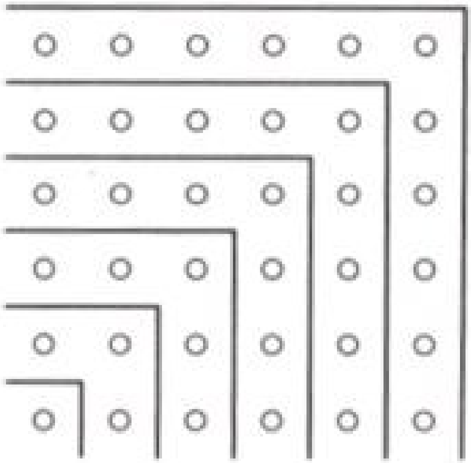
Link your ideas



$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots = 1$$

THEOREM: $1+3+5+\ldots+(2n-1)=n^2$

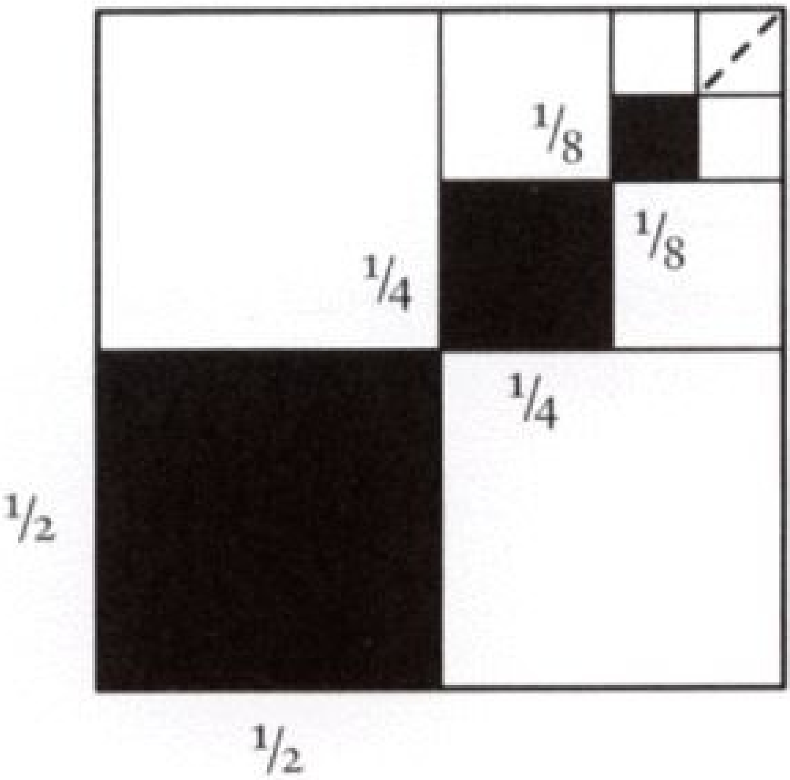
PROOF:



THEOREM:

$$\sum_{n=1}^{\infty}\frac{1}{4^n}=\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\cdots=\frac{1}{3}$$

PROOF:



James Robert Brown

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ \dots}}}}=2$$

$$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ \dots}}}}=3$$

$$\sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12+\ \dots}}}}=4$$

$$\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20+\ \dots}}}}=5$$

$$\sqrt{30+\sqrt{30+\sqrt{30+\sqrt{30+\ \dots}}}}=6$$

$$\sqrt{42+\sqrt{42+\sqrt{42+\sqrt{42+\ \dots}}}}=7$$

$$\sqrt{56+\sqrt{56+\sqrt{56+\sqrt{56+\ \dots}}}}=8$$

$$\sqrt{72+\sqrt{72+\sqrt{72+\sqrt{72+\ \dots}}}}=9$$

$$\sqrt{90+\sqrt{90+\sqrt{90+\sqrt{90+\ \dots}}}}=10$$

$$\sqrt{110+\sqrt{110+\sqrt{110+\sqrt{110+\ \dots}}}}=11$$

$$\sqrt{132+\sqrt{132+\sqrt{132+\sqrt{132+\ \dots}}}}=12$$

$$\sqrt{156+\sqrt{156+\sqrt{156+\sqrt{156+\ \dots}}}}=13$$

$$\sqrt{182+\sqrt{182+\sqrt{182+\sqrt{182+\ \dots}}}}=14$$

$$\sqrt{210+\sqrt{210+\sqrt{210+\sqrt{210+\ \dots}}}}=15$$

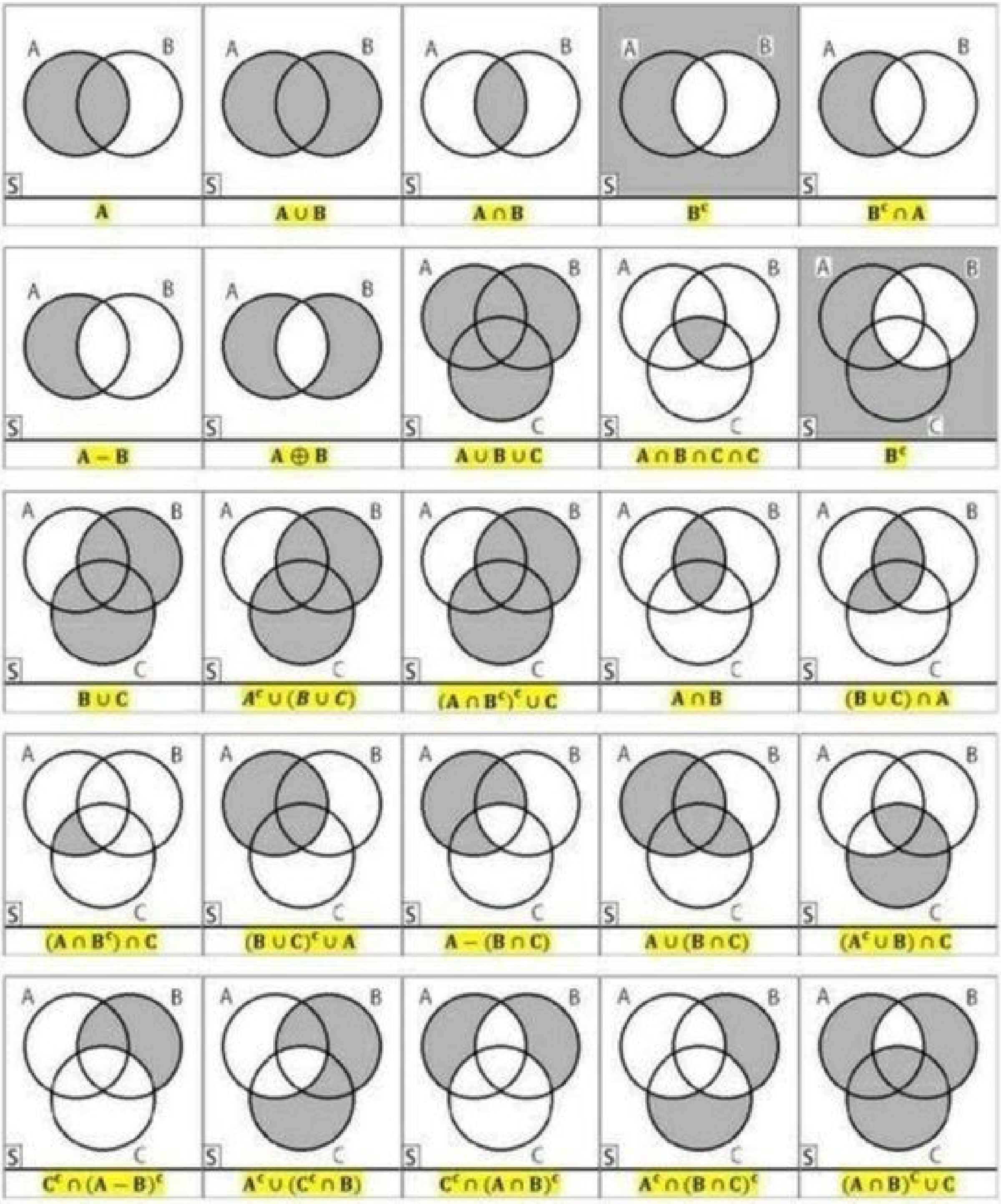
...

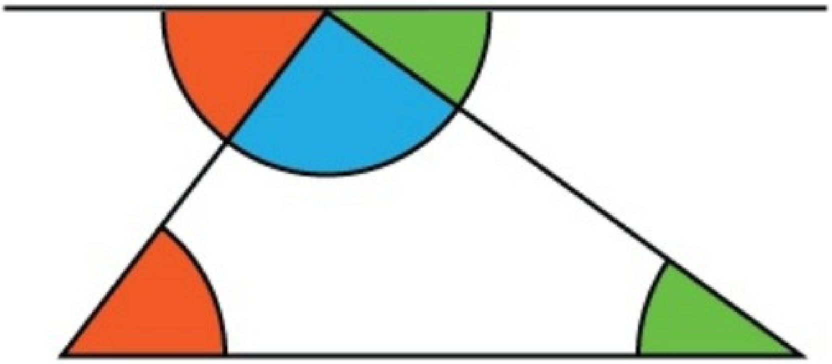
$$\sqrt{n(n-1)+\sqrt{n(n-1)+\sqrt{n(n-1)+\sqrt{n(n-1)+\ \dots}}}}=n$$

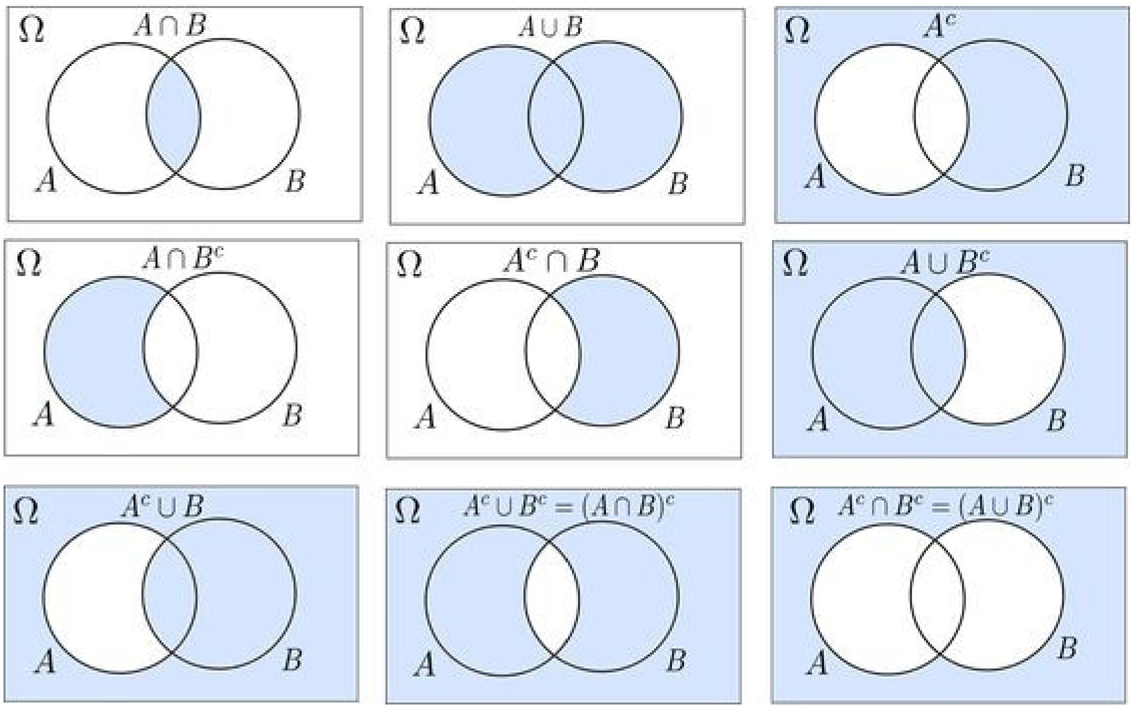
$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\dots}}}}}}}}}}}}}}}}$$

is the same as

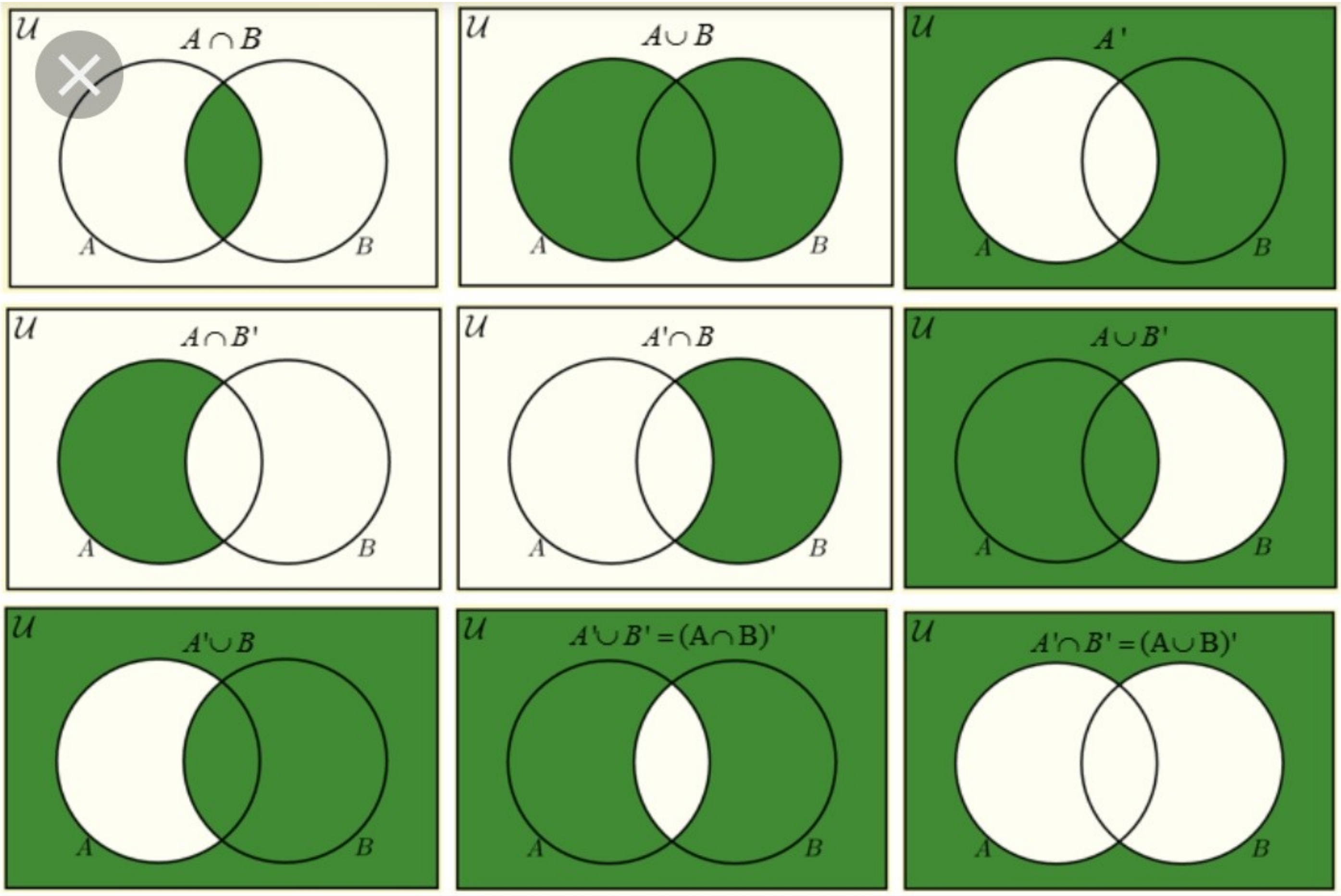
$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\dots}}}}}}}$$


A
$A \cup B$
$A \cap B$
B^c
$B^c \cap A$
$A - B$
$A \oplus B$
$A \cup B \cup C$
$A \cap B \cap C \cap C$
B^c
$B \cup C$
$A^c \cup (B \cup C)$
$(A \cap B^c)^c \cup C$
$A \cap B$
$(B \cup C) \cap A$
$(A \cap B^c) \cap C$
$(B \cup C)^c \cup A$
$A - (B \cap C)$
$A \cup (B \cap C)$
$(A^c \cup B) \cap C$
$C^c \cap (A - B)^c$
$A^c \cup (C^c \cap B)$
$C^c \cap (A \cap B)^c$
$A^c \cap (B \cap C)^c$
$(A \cap B)^C \cup C$

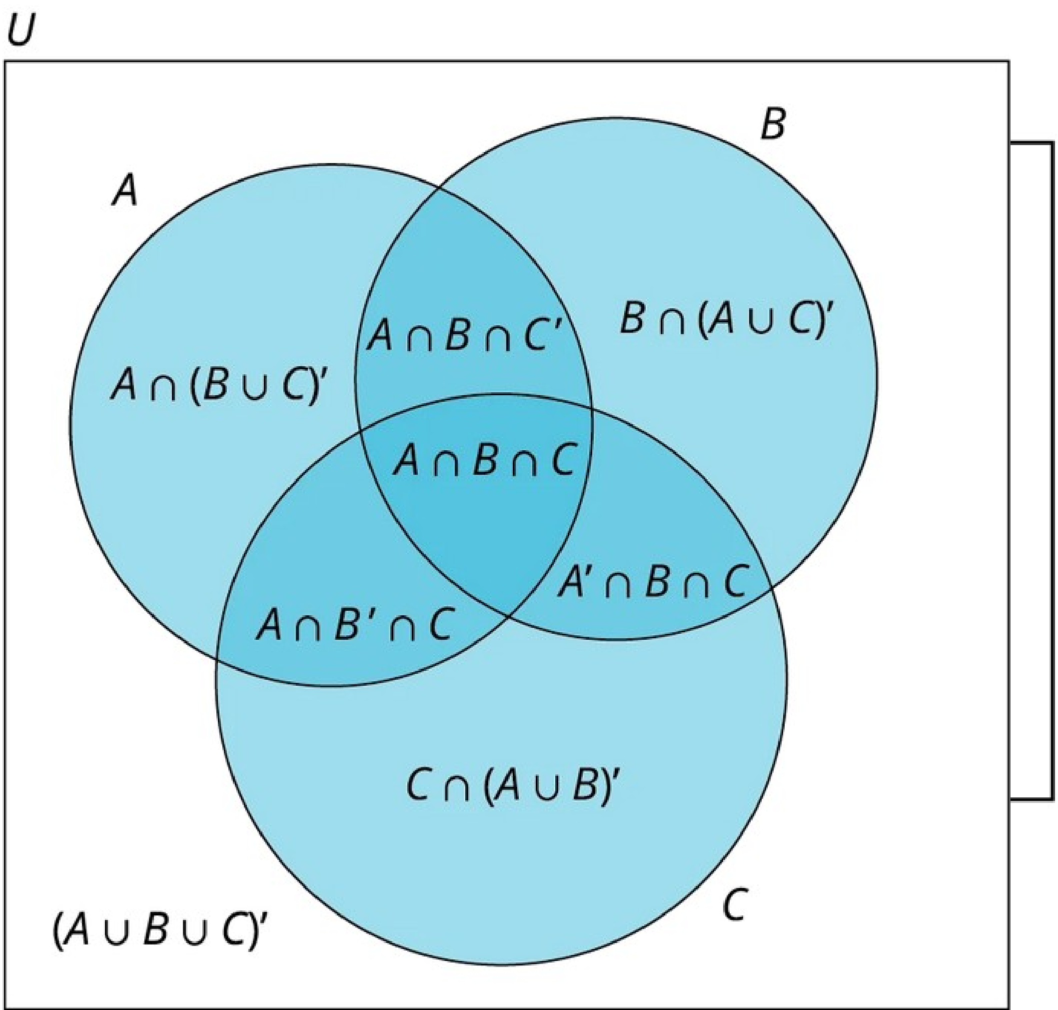




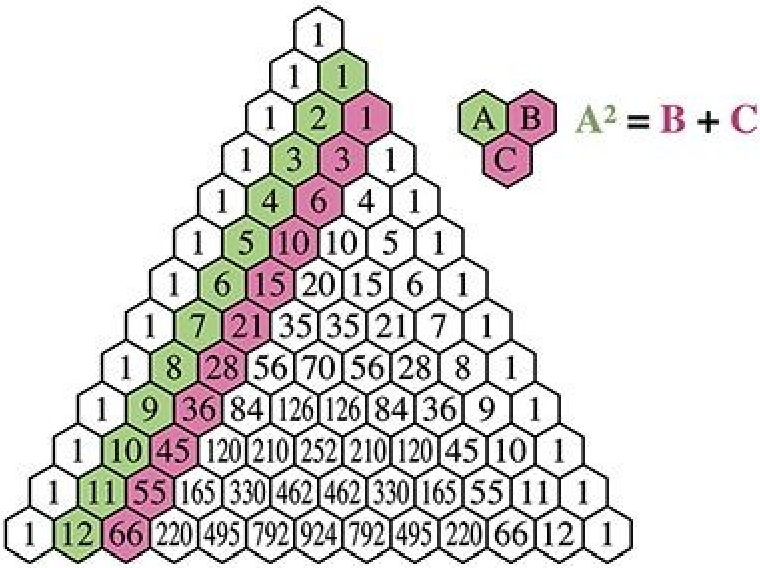
Ω
$A \cap B$
A
B
Ω
$A \cup B$
A
B
Ω
A^c
A
B
Ω
$A \cap B^c$
A
B
Ω
$A^c \cap B$
A
B
Ω
$A \cup B^c$
A
B
Ω
$A^c \cup B$
A
B
Ω
$A^c \cup B^c = (A \cap B)^c$
A
B
Ω
$A^c \cap B^c = (A \cup B)^c$
A
B

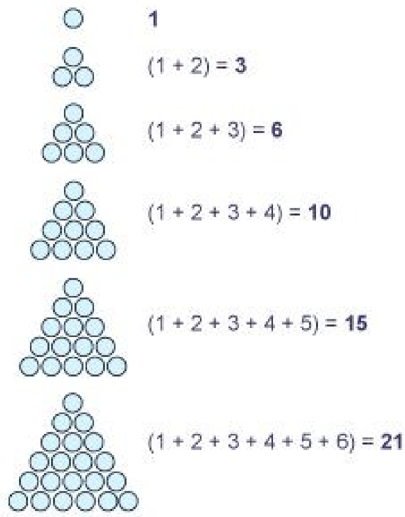


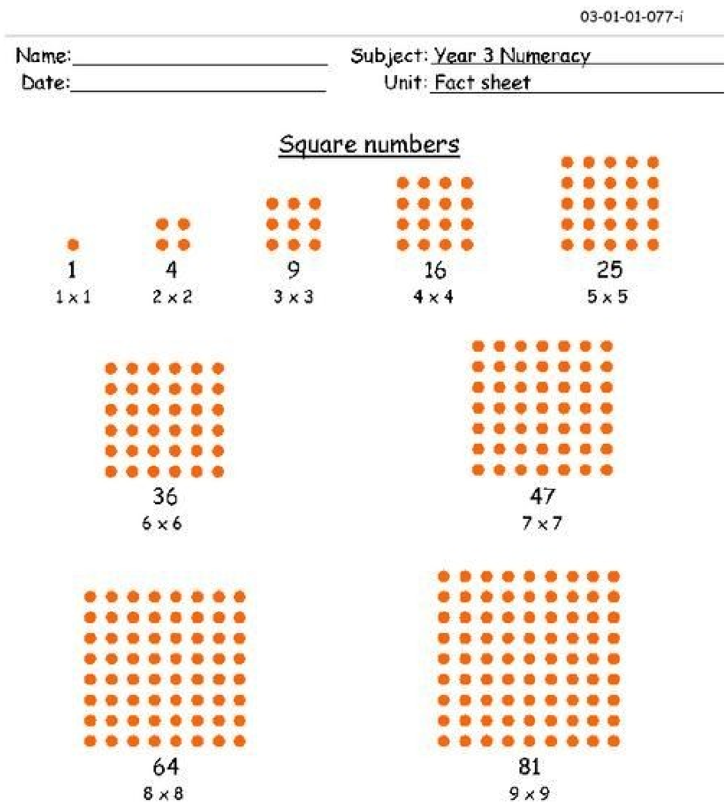
A ∩ B
A ∪ B
A'
A ∩ B'
A' ∩ B
A ∪ B'
A' ∪ B
A' ∪ B' = (A ∩ B)'
A' ∩ B' = (A ∪ B)'

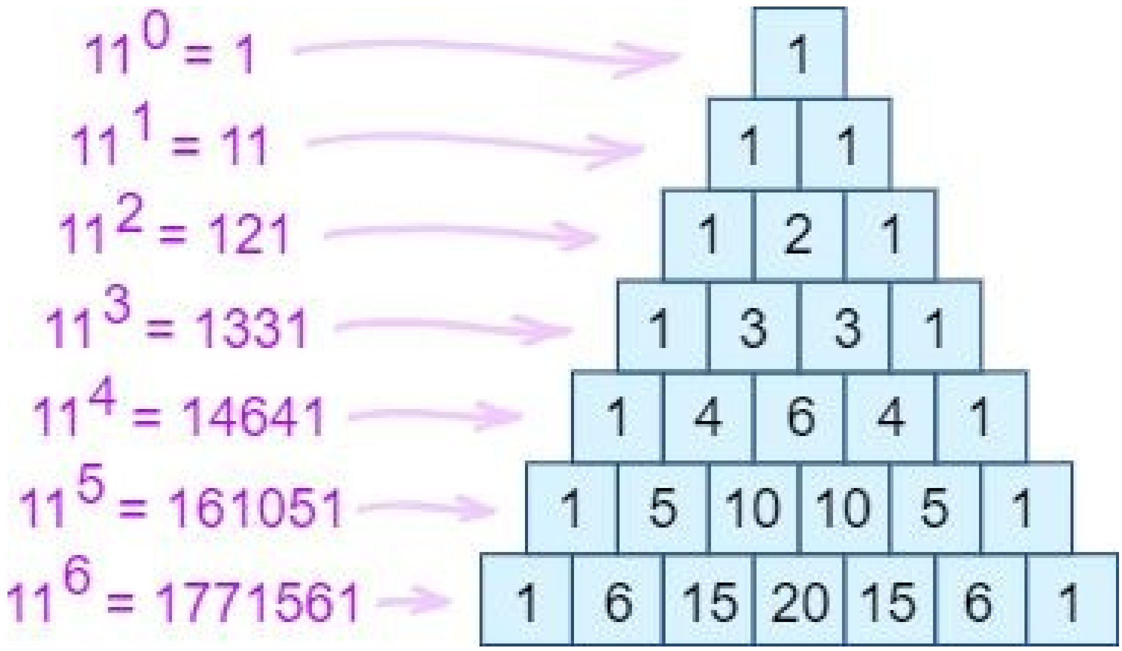


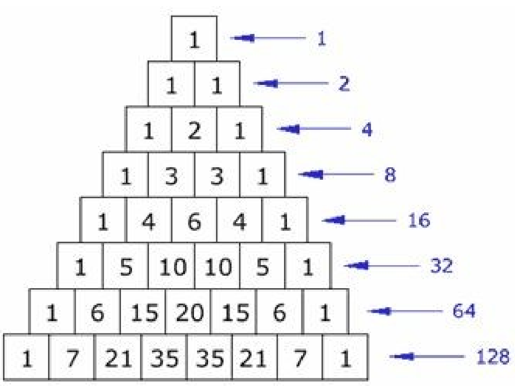
U
A
B
C
A ∩ (B ∪ C)'
A ∩ B ∩ C'
B ∩ (A ∪ C)'
A ∩ B ∩ C
A' ∩ B ∩ C
A ∩ B' ∩ C
C ∩ (A ∪ B)'
(A ∪ B ∪ C)'

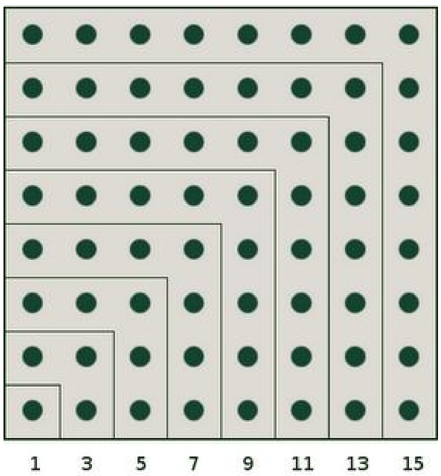

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
A B C
A² = B + C

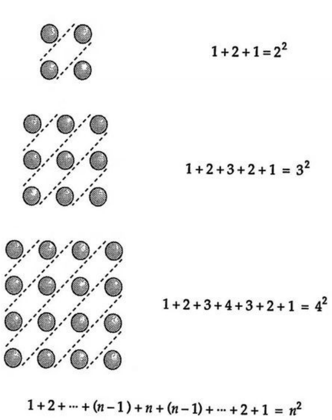

1
(1 + 2) = 3
(1 + 2 + 3) = 6
(1 + 2 + 3 + 4) = 10
(1 + 2 + 3 + 4 + 5) = 15
(1 + 2 + 3 + 4 + 5 + 6) = 21


03-01-01-077-i
Name:
Subject: Year 3 Numeracy
Date:
Unit: Fact sheet
Square numbers
1
1 x 1
4
2 x 2
9
3 x 3
16
4 x 4
25
5 x 5
36
6 x 6
47
7 x 7
64
8 x 8
81
9 x 9

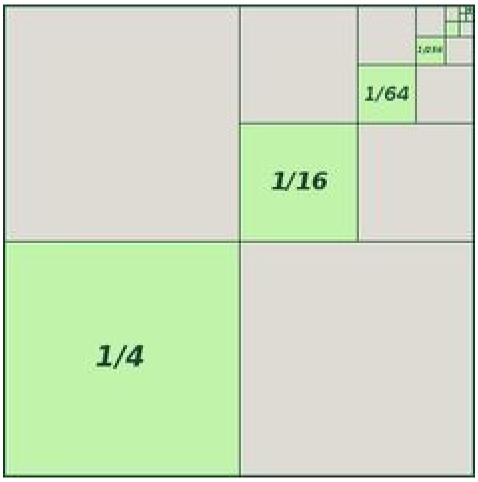


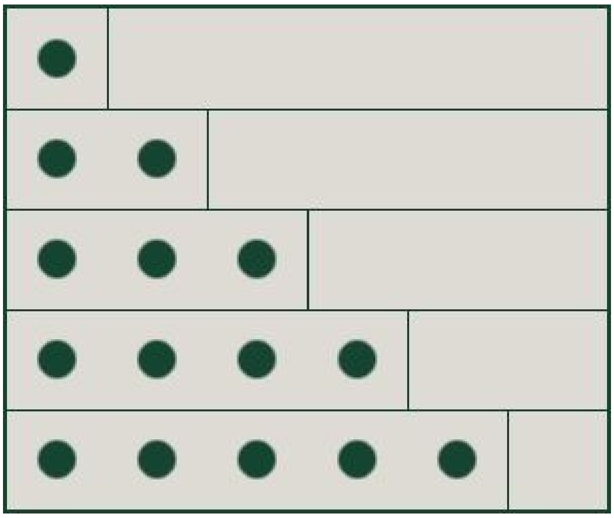


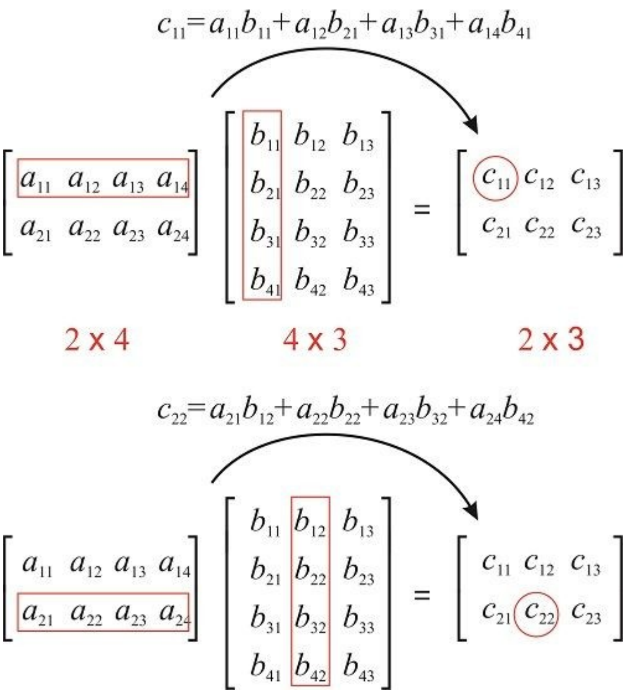




—"The ancient Greeks"
(as cited by Martin Gardner)

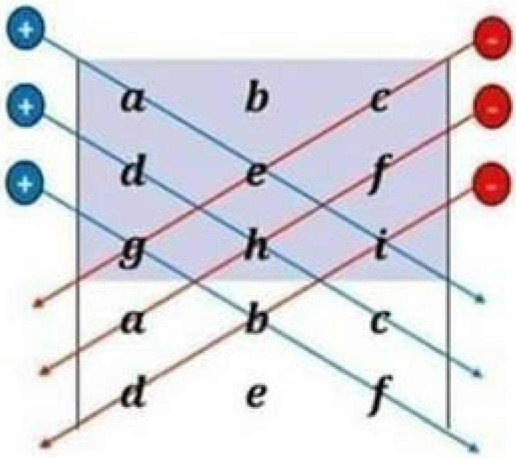

1/4
1/16
1/64




$c_{11}=a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31}+a_{14}b_{41}$
2 x 4
4 x 3
2 x 3
$c_{22}=a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32}+a_{24}b_{42}$

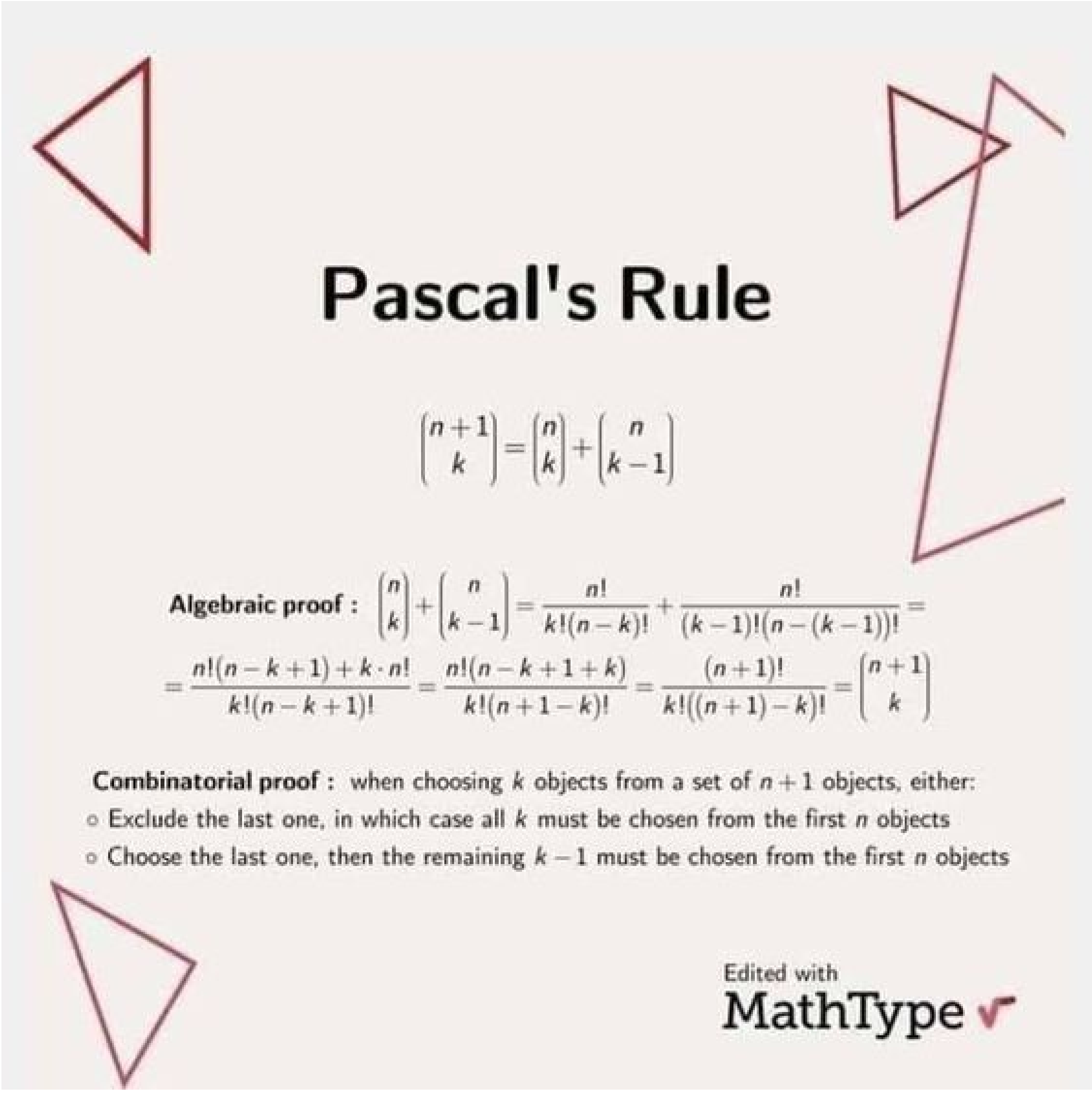
SARRUS RULE

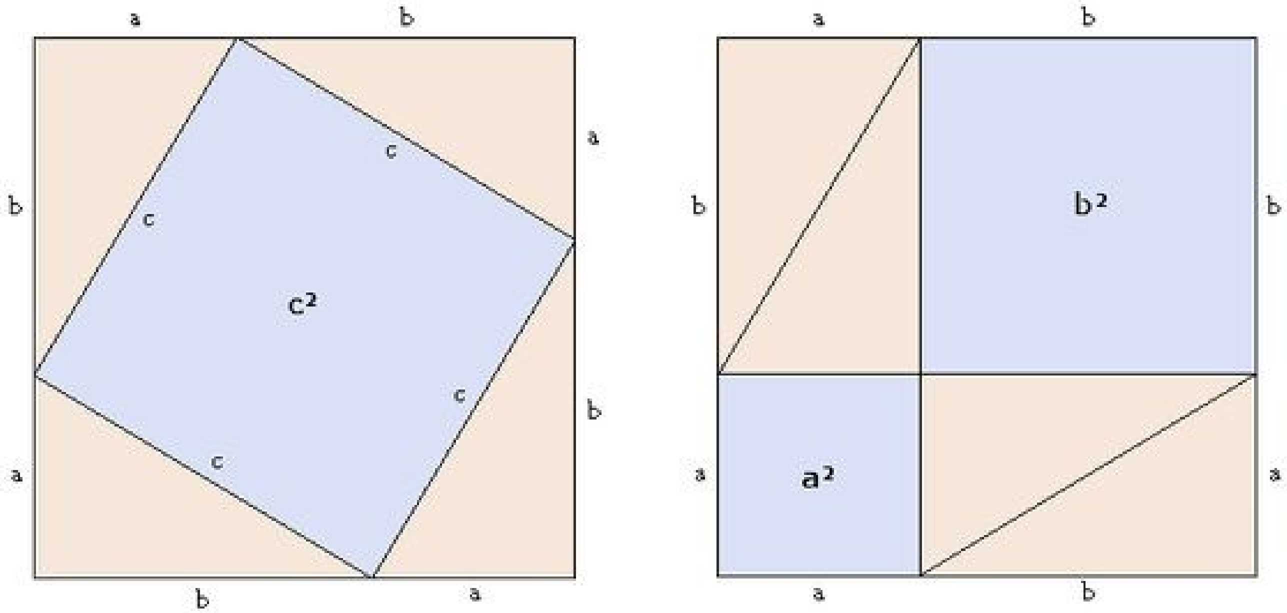
If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ a & b & c \\ d & e & f \end{bmatrix}$

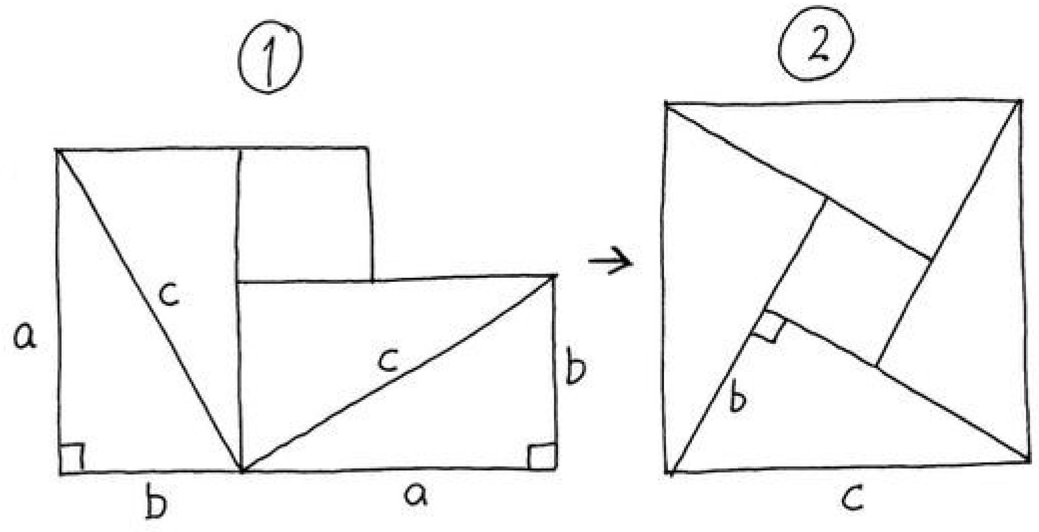


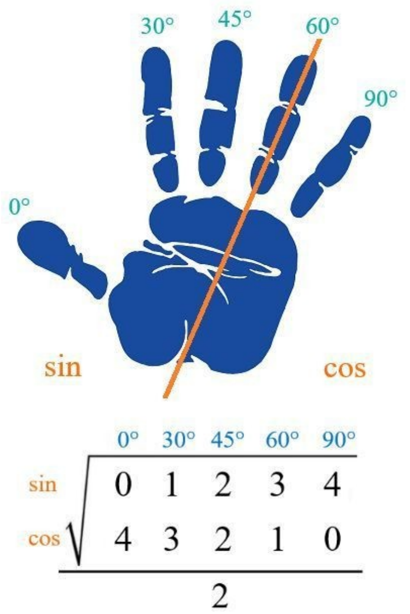
$$\det A = aei + dhc + gbf - ceg - fha - ibd$$

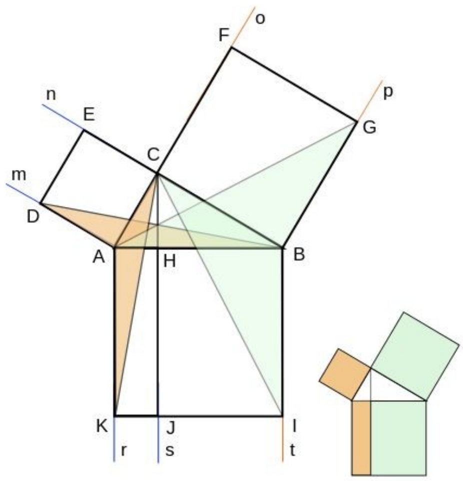
Only applies to 3×3 matrices.

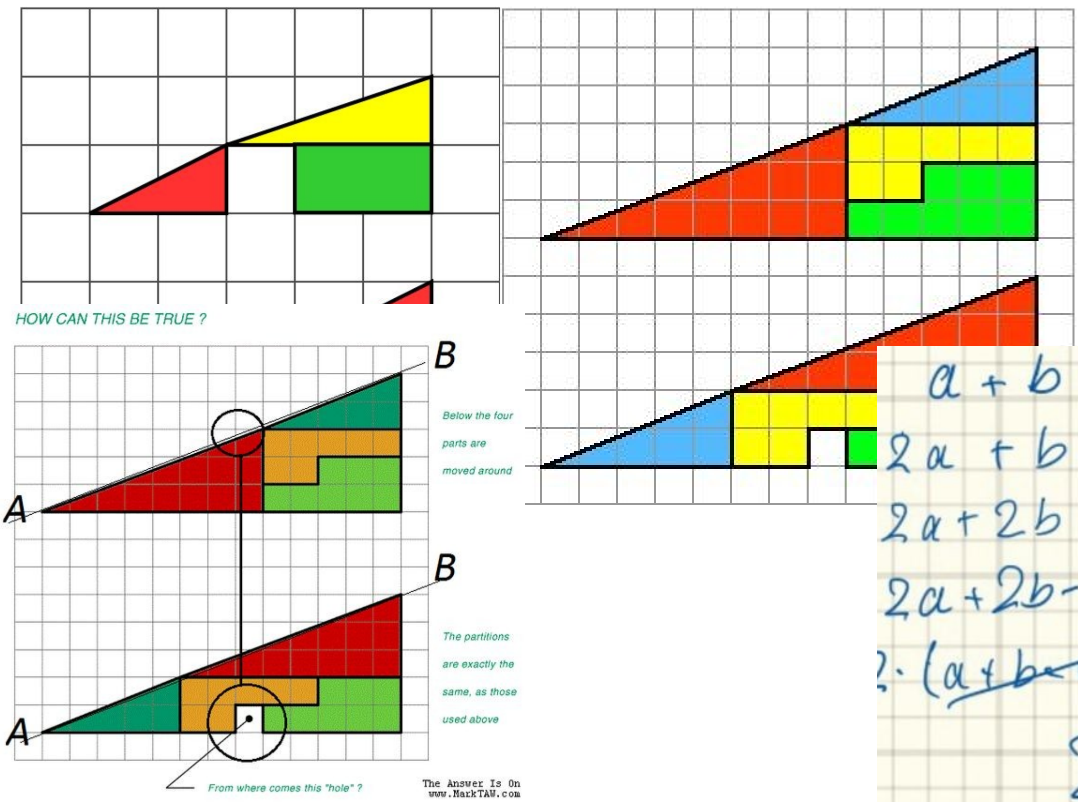

Pascal's Rule
$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$
Algebraic proof : $\binom{n}{k} + \binom{n}{k-1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-(k-1))!} =$
$= \frac{n!(n-k+1)+k \cdot n!}{k!(n-k+1)!} = \frac{n!(n-k+1+k)}{k!(n+1-k)!} = \frac{(n+1)!}{k!((n+1)-k)!} = \binom{n+1}{k}$
Combinatorial proof : when choosing k objects from a set of $n+1$ objects, either:
∘ Exclude the last one, in which case all k must be chosen from the first n objects
∘ Choose the last one, then the remaining $k-1$ must be chosen from the first n objects
Edited with
MathType

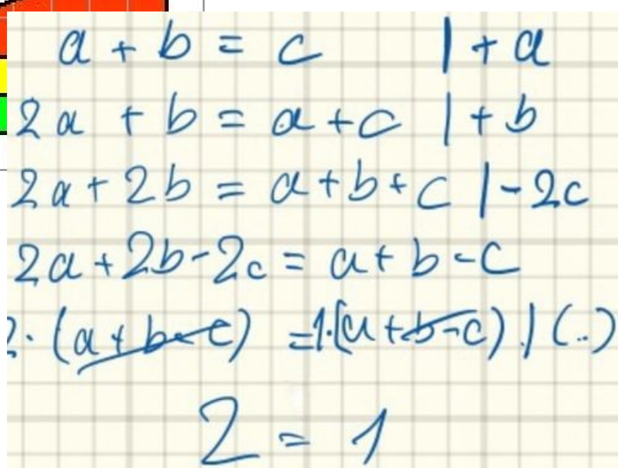

a
b
c
c²
a²
b²

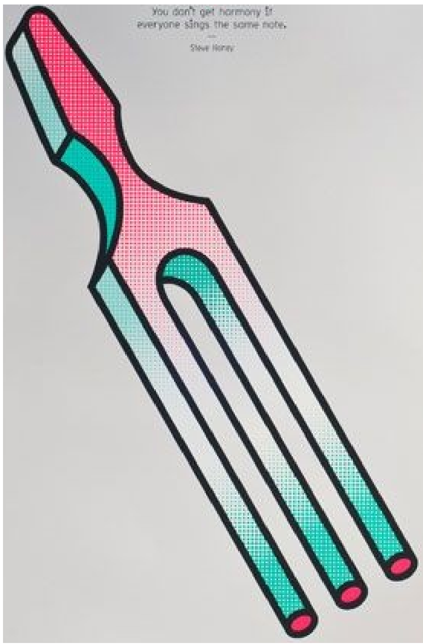

1
2
a
c
c
b
b
a
b
c

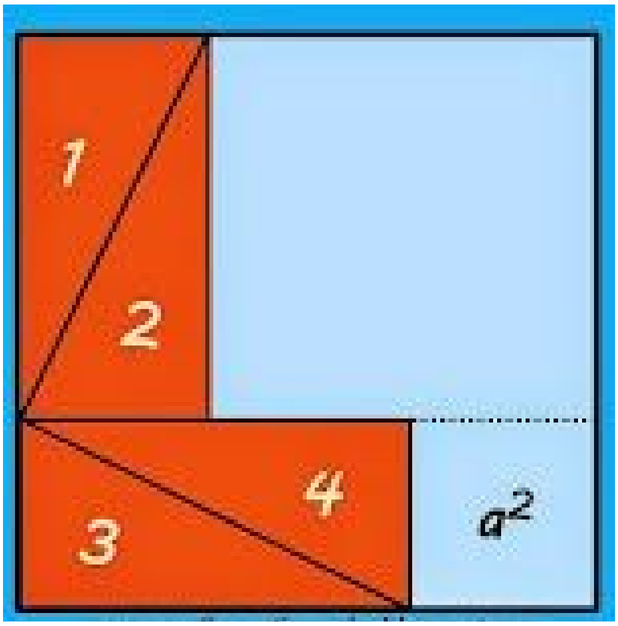

30°
45°
60°
90°
0°
sin
cos
0° 30° 45° 60° 90°
sin 0 1 2 3 4
cos 4 3 2 1 0
2


o
F
n
E
p
G
C
m
D
A
H
B
K
J
I
r
s
t


HOW CAN THIS BE TRUE ?
B
A
Below the four
parts are
moved around
B
A
The partitions
are exactly the
same, as those
used above
From where comes this "hole" ?
The Answer Is On
www.MarkTAW.com


a + b = c | + a
2a + b = a + c | + b
2a + 2b = a + b + c | - 2c
2a + 2b - 2c = a + b - c
2 · (a + b - c) = 1 · (a + b - c) | (..)
2 = 1



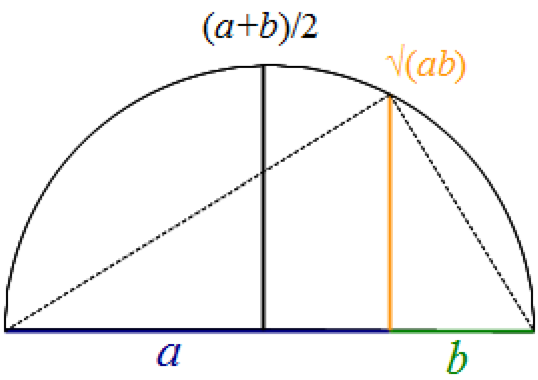


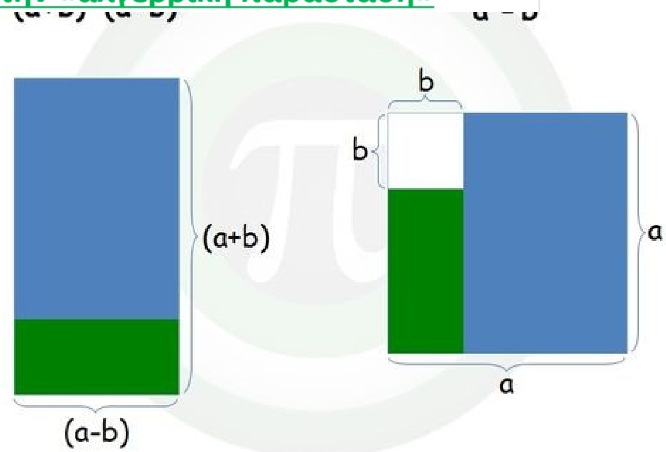
Πώς θα μάθω <u>να εφαρμόζω σωστά</u> την ταυτότητα $(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$ και κάθε ταυτότητα;

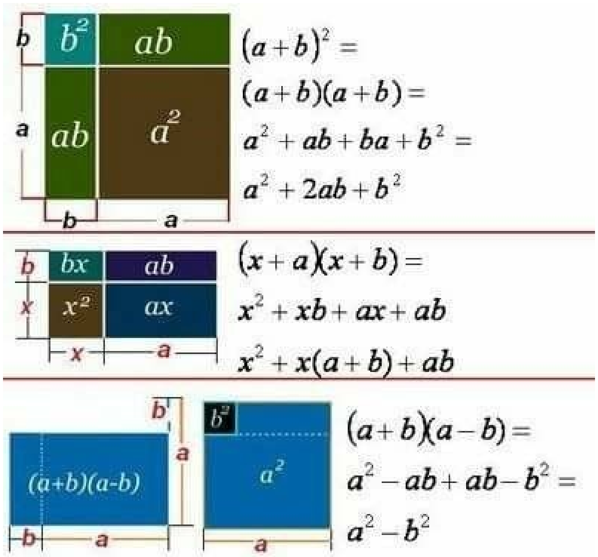
<u>Απάντηση</u> : «Το τετράγωνο αθροίσματος ισούται με το τετράγωνο του πρώτου και το διπλάσιο του πρώτου επί το δεύτερο, και το τετράγωνο του δεύτερου»

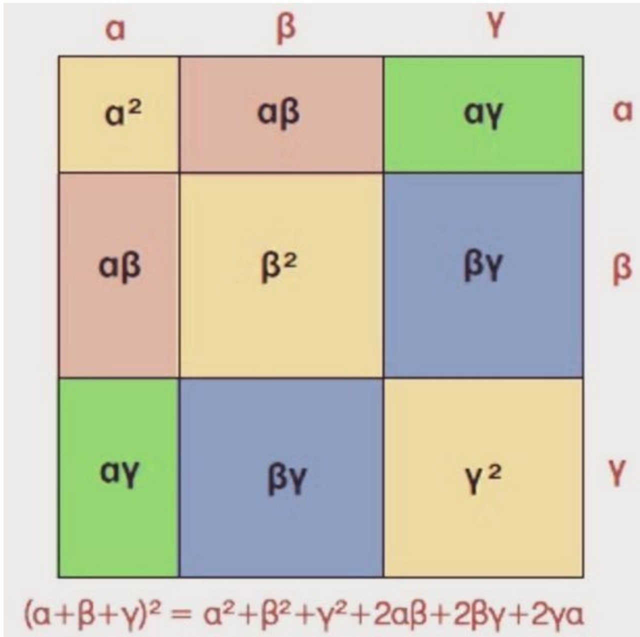
Μόνον έτσι, δεν υπάρχει άλλος τρόπος, <u>ας νομίζεις αλλιώς!!!</u>

Το μαθαίνεις «απ΄ έξω», **<u>«κοιτάζοντας» τα λόγια, πάνω στην «αλγεβρική παράσταση»</u>**

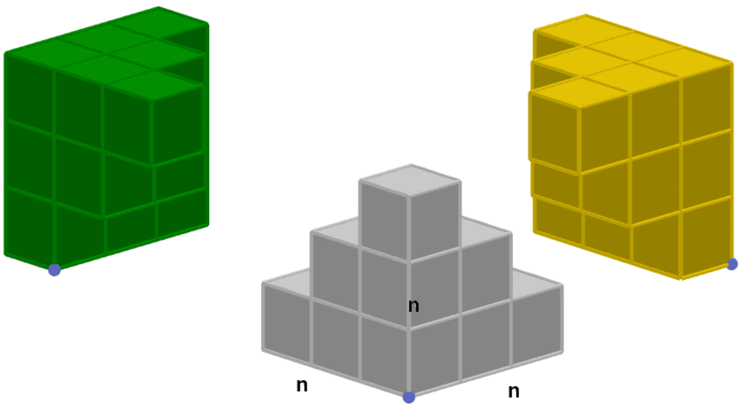


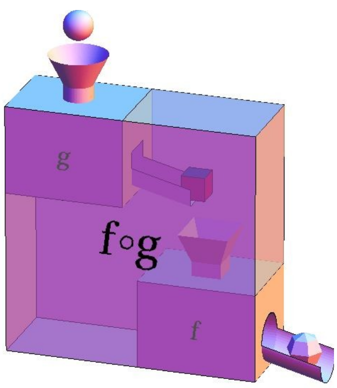


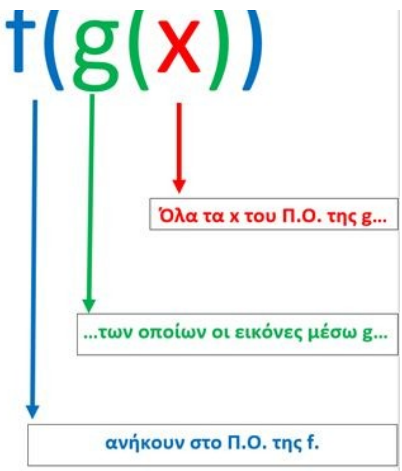


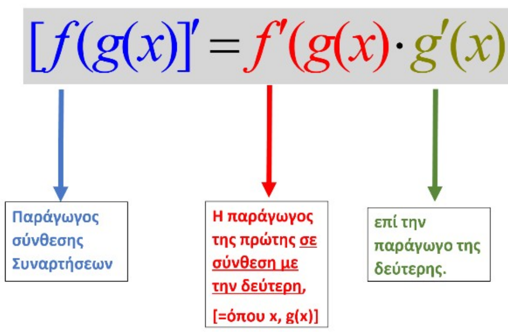


$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{3}n(n+1)(n+\frac{1}{2})$$









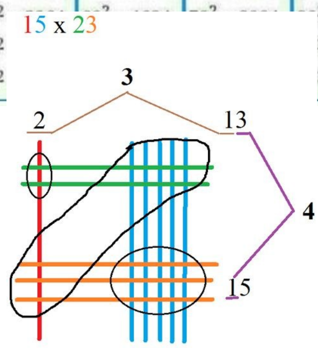
$$
\begin{aligned}
3 &= \sqrt{9} \\
&= \sqrt{1+8} \\
&= \sqrt{1+2\cdot 4} \\
&= \sqrt{1+2\sqrt{16}} \\
&= \sqrt{1+2\sqrt{1+15}} \\
&= \sqrt{1+2\sqrt{1+3\cdot 5}} \\
&= \sqrt{1+2\sqrt{1+3\sqrt{25}}} \\
&= \sqrt{1+2\sqrt{1+3\sqrt{1+4\cdot 6}}} \\
&= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+\dots}}}}
\end{aligned}
$$

PRIME NUMBERS
BETWEEN 1 AND 1,000

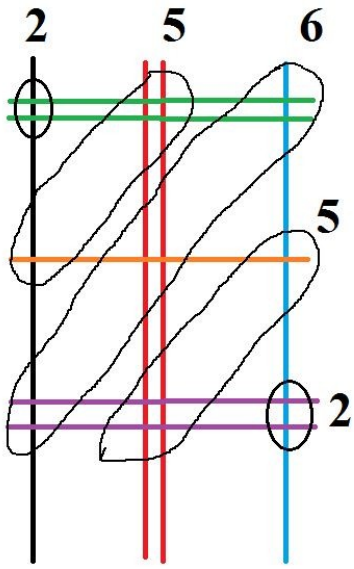
2	79	191	311	439	577	709	857
3	83	193	313	443	587	719	859
5	89	197	317	449	593	727	863
7	97	199	331	457	599	733	877
11	101	211	337	461	601	739	881
13	103	223	347	463	607	743	883
17	107	227	349	467	613	751	887
19	109	229	353	479	617	757	907
23	113	233	359	487	619	761	911
29	127	239	367	491	631	769	919
31	131	241	373	499	641	773	929
37	137	251	379	503	643	787	937
41	139	257	383	509	647	797	941
43	149	263	389	521	653	809	947
47	151	269	397	523	659	811	953
53	157	271	401	541	661	821	967
59	163	277	409	547	673	823	971
61	167	281	419	557	677	827	977
67	173	283	421	563	683	829	983
71	179	293	431	569	691	839	991
73	181	307	433	571	701	853	997

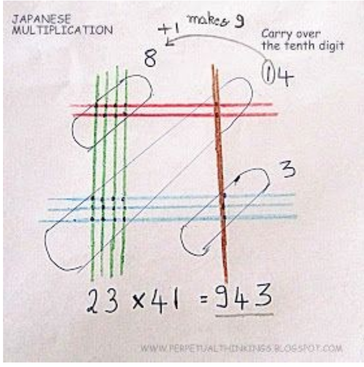
Table of Squares From 1 to 100

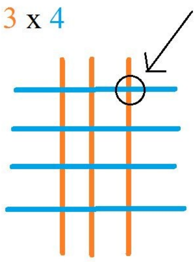
$1^2 = 1$	$11^2 = 121$	$21^2 = 441$	$31^2 = 961$	$41^2 = 1681$
$2^2 = 4$	$12^2 = 144$	$22^2 = 484$	$32^2 = 1024$	$42^2 = 1764$
$3^2 = 9$	$13^2 = 169$	$23^2 = 529$	$33^2 = 1089$	$43^2 = 1849$
$4^2 = 16$	$14^2 = 196$	$24^2 = 576$	$34^2 = 1156$	$44^2 = 1936$
$5^2 = 25$	$15^2 = 225$	$25^2 = 625$	$35^2 = 1225$	$45^2 = 2025$
$6^2 = 36$	$16^2 = 256$	$26^2 = 676$	$36^2 = 1296$	$46^2 = 2116$
$7^2 = 49$	$17^2 = 289$	$27^2 = 729$	$37^2 = 1369$	$47^2 = 2209$
$8^2 = 64$	$18^2 = 324$	$28^2 = 784$	$38^2 = 1444$	$48^2 = 2304$
$9^2 = 81$	$19^2 = 361$	$29^2 = 841$	$39^2 = 1521$	$49^2 = 2401$
$10^2 = 100$	$20^2 = 400$	$30^2 = 900$	$40^2 = 1600$	$50^2 = 2500$
$51^2 = 2601$	$61^2 = 3721$	$71^2 = 5041$	$81^2 = 6561$	$91^2 = 8281$
$52^2 = 2704$	$62^2 = 3844$	$72^2 = 5184$	$82^2 = 6724$	$92^2 = 8464$
$53^2 = 2809$	$63^2 = 3969$	$73^2 = 5329$	$83^2 = 6889$	$93^2 = 8649$
$54^2 = 2916$	$64^2 = 4096$	$74^2 = 5476$	$84^2 = 7056$	$94^2 = 8836$
$55^2 = 3025$	$65^2 = 4225$	$75^2 = 5625$	$85^2 = 7225$	$95^2 = 9025$
$56^2 = 3136$	$66^2 = 4356$	$76^2 = 5776$	$86^2 = 7396$	$96^2 = 9216$
$57^2 = 3249$	$67^2 = 4489$	$77^2 = 5929$	$87^2 = 7569$	$97^2 = 9409$
58^2			$^2 = 7744$	$98^2 = 9604$
59^2			$^2 = 7921$	$99^2 = 9801$
60^2			$^2 = 8100$	$100^2 = 10000$

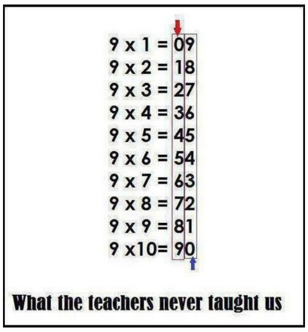


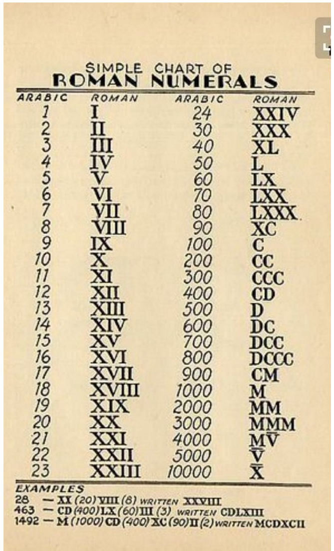
121 x 212 =25652









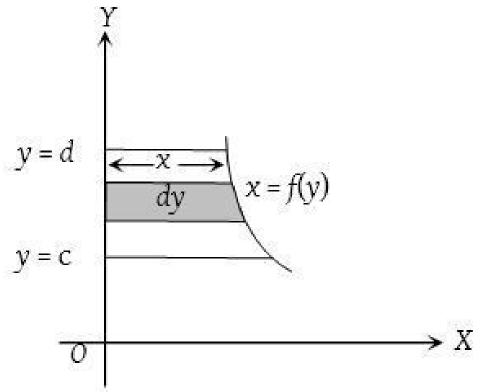


SIMPLE CHART OF ROMAN NUMERALS

ARABIC	ROMAN	ARABIC	ROMAN
1	I	24	XXIV
2	II	30	XXX
3	III	40	XL
4	IV	50	L
5	V	60	LX
6	VI	70	LXX
7	VII	80	LXXX
8	VIII	90	XC
9	IX	100	C
10	X	200	CC
11	XI	300	CCC
12	XII	400	CD
13	XIII	500	D
14	XIV	600	DC
15	XV	700	DCC
16	XVI	800	DCCC
17	XVII	900	CM
18	XVIII	1000	M
19	XIX	2000	MM
20	XX	3000	MMM
21	XXI	4000	$M\bar{V}$
22	XXII	5000	$\bar{V}$
23	XXIII	10000	$\bar{X}$

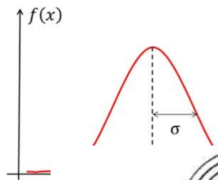
EXAMPLES
28 — XX (20) VIII (8) WRITTEN XXVIII
463 — CD (400) LX (60) III (3) WRITTEN CDLXIII
1492 — M (1000) CD (400) XC (90) II (2) WRITTEN MCDXCII

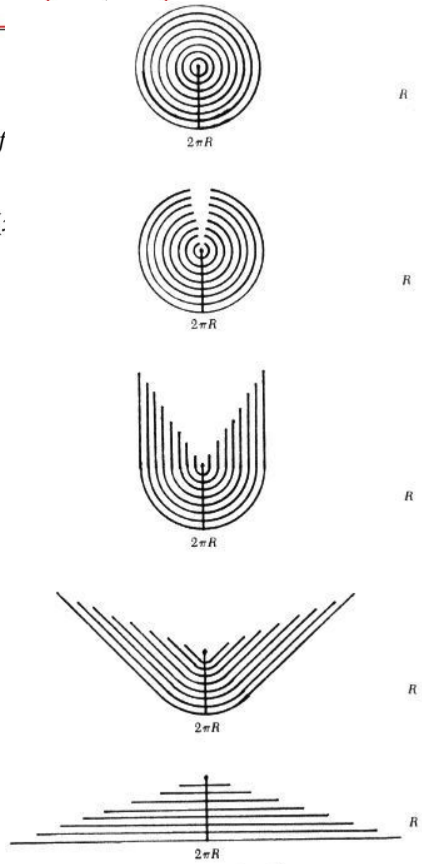
$$\text{Area} = \int_c^d x\,dy = \int_c^d f(y)dy$$



NORMAL DISTRIBUTION (GAUSSIAN DISTRIBUTION)

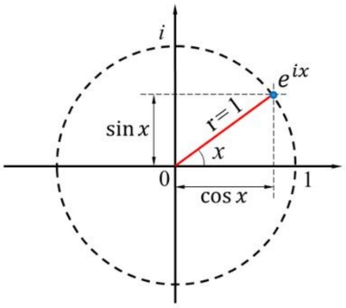
f: density function, F: Distribution function, μ: mean



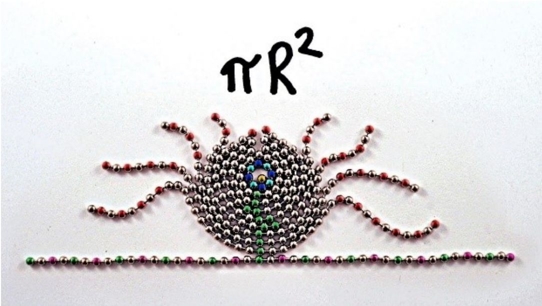


$A = (1/2)(2\pi R)R = \pi R^2$

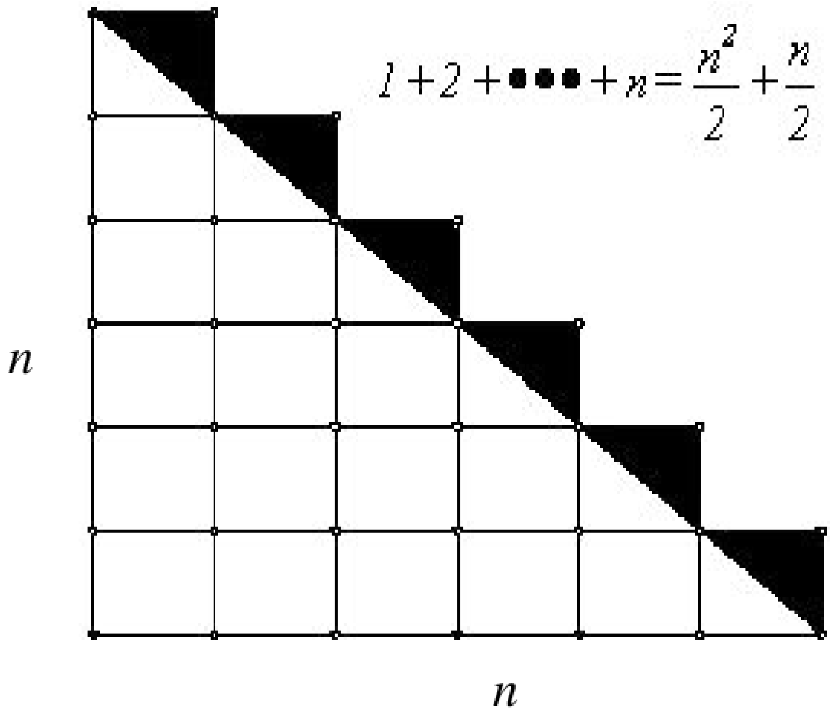
$$e^{ix} = \cos x + i \sin x$$



$$e^{i\pi} = \cos\pi + i\sin\pi$$
$$e^{i\pi} = -1 + i \times 0$$
$$e^{i\pi} = -1$$
$$e^{i\pi} + 1 = 0$$

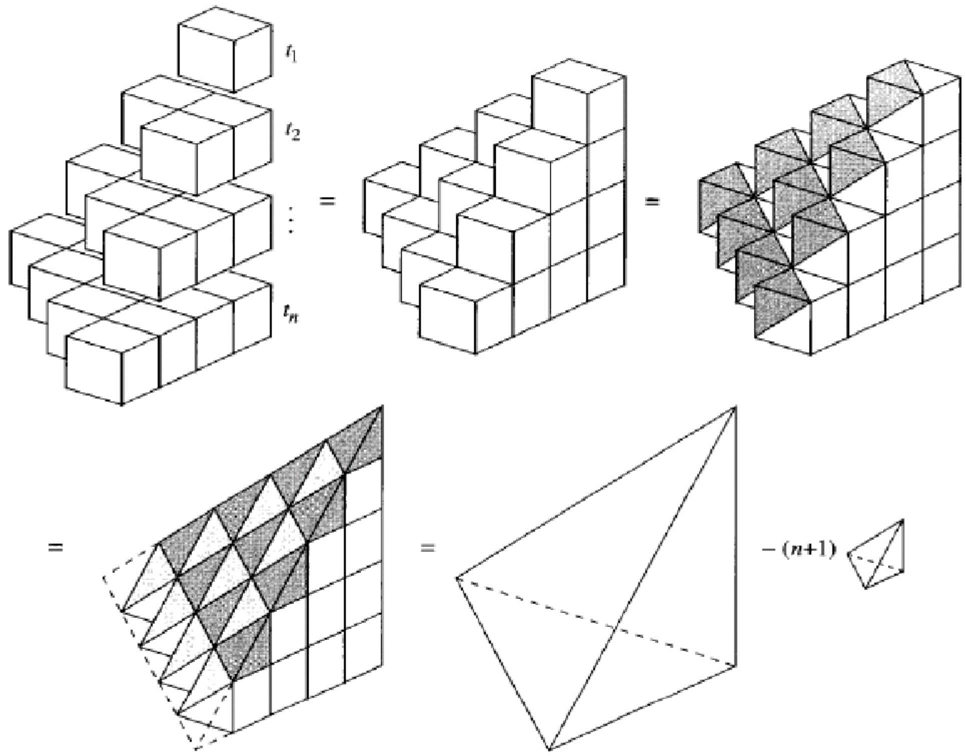


πR^2



$1+2+\bullet\bullet\bullet+n=\frac{n^2}{2}+\frac{n}{2}$
n
n

$$t_n = 1 + 2 + \cdots + n \Rightarrow t_1 + t_2 + \cdots + t_n = \frac{n(n+1)(n+2)}{6}$$



$$t_1 + t_2 + \cdots + t_n = \frac{1}{6}(n+1)^3 - (n+1)\cdot\frac{1}{6} = \frac{n(n+1)(n+2)}{6}$$

	1	**2**	**3**	.	.	.	***n***
+	**2**	**4**	**6**	.	.	.	**2*n***
+	**3**	**6**	**9**	.	.	.	**3*n***
+	.	.	.	.	.	.	.
	.	.	.	.	.		.
	.	.	.	.	.	.	.
+	***n***	**2*n***	**3*n***	.	.	.	n^2

$$= \sum_{i=1}^{n} i + 2\sum_{i=1}^{n} i + 3\sum_{i=1}^{n} i + \ldots + n\sum_{i=1}^{n} i$$

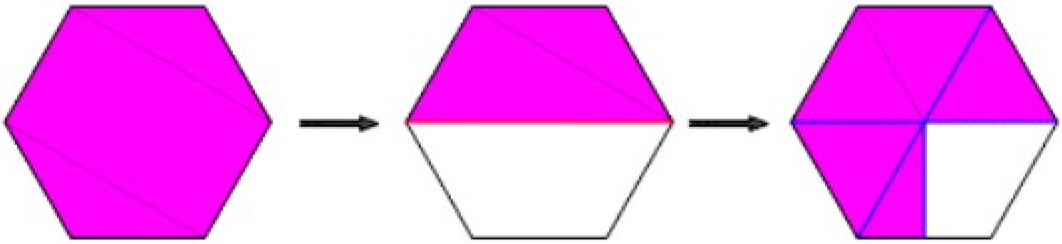
$$= (\sum_{i=1}^{n} i)^2$$

$$= \left(\frac{n(n+1)}{2}\right)^2$$

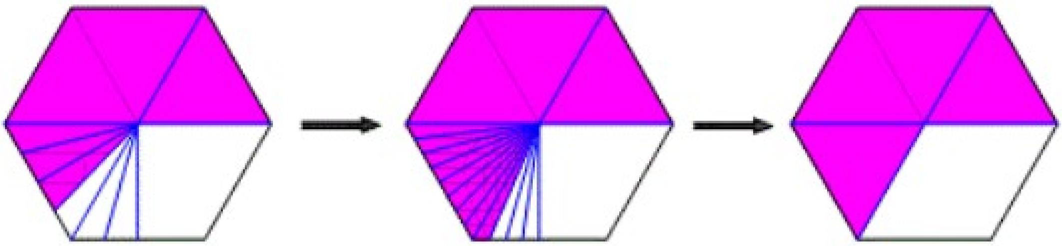
	1	**2**	**3**	.	.	.	***n***
+	**2**	**4**	**6**	.	.	.	**2*n***
+	**3**	**6**	**9**	.	.	.	**3*n***
+	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
+	***n***	**2*n***	**3*n***	.	.	.	n^2

$$= 1(1^2) + 2(2)^2 + \ldots + n(n)^2$$

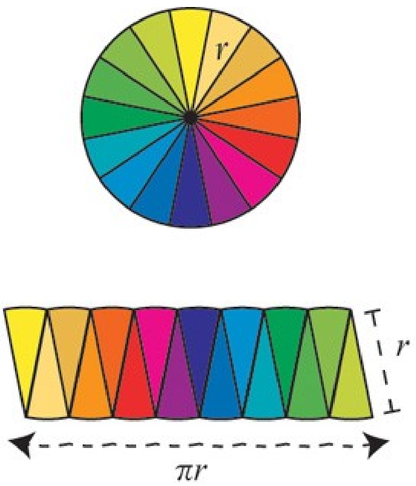
$$= \sum_{i=1}^{n} i^3$$

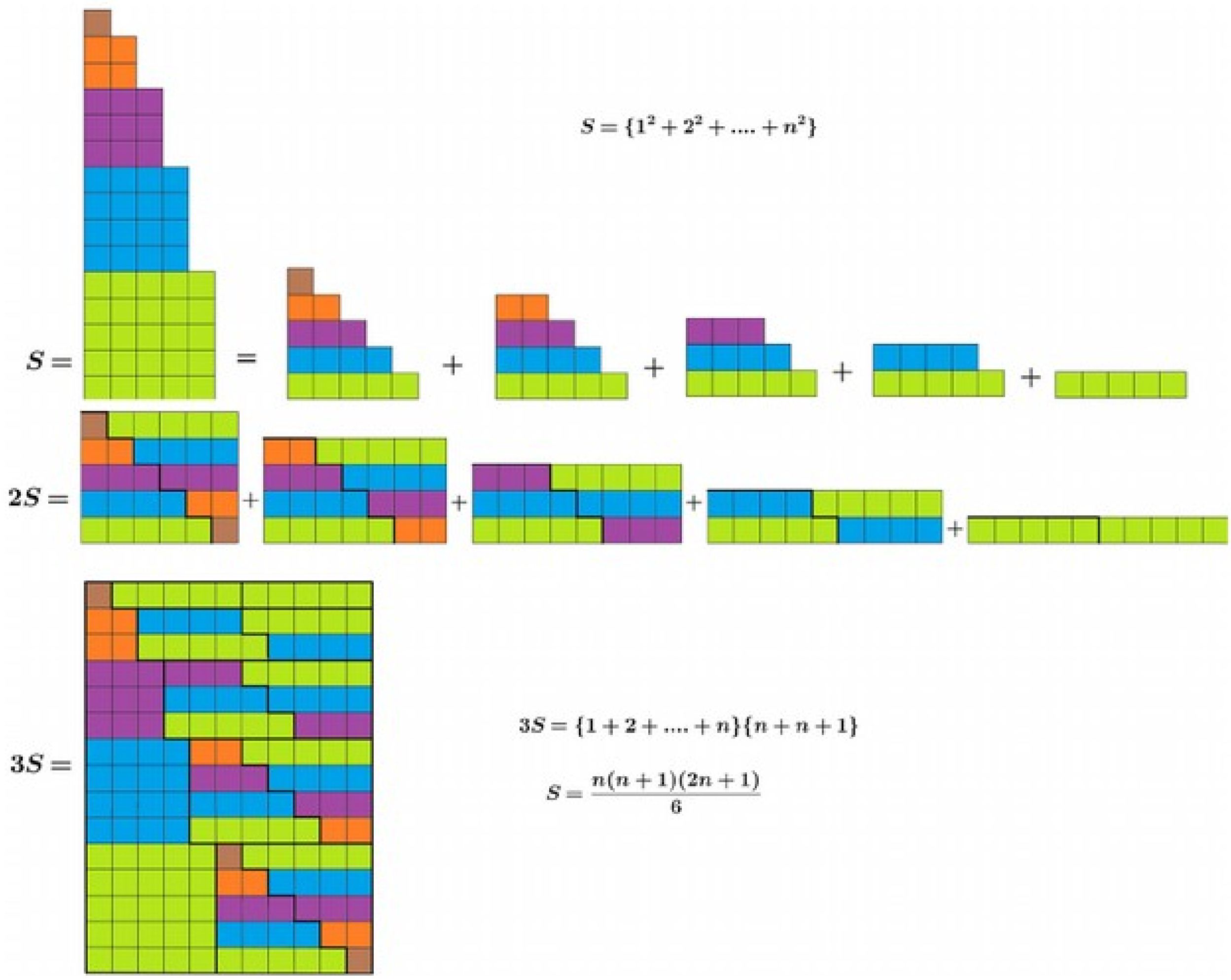


1 $\qquad$ $1-\frac{1}{2}$ $\qquad$ $1-\frac{1}{2}+\frac{1}{4}$

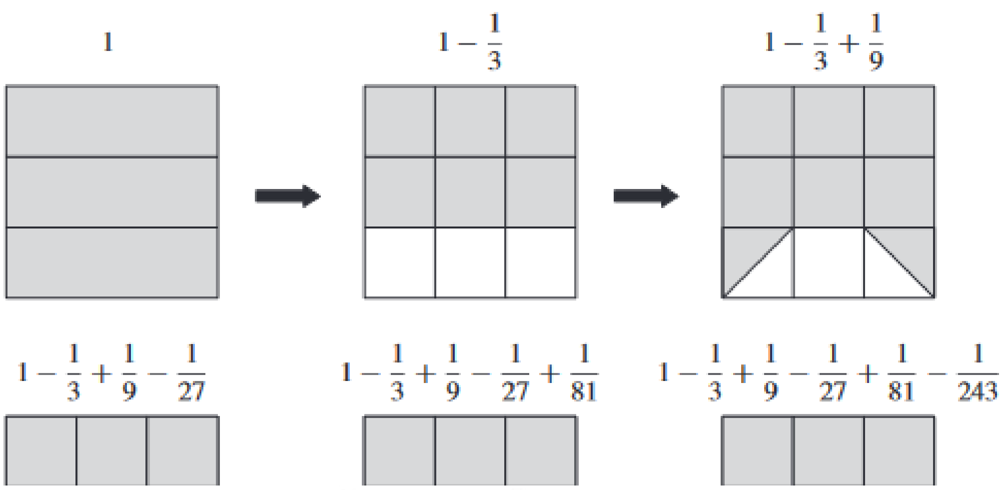


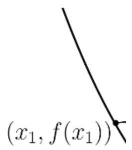
$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}$ $\qquad$ $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}$ $\qquad$ $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}\cdots\cdots=\frac{2}{3}$

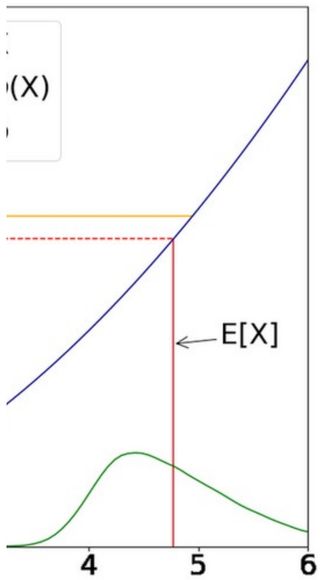



$S = \{1^2 + 2^2 + ... + n^2\}$
$S =$
$=$
$+$
$2S =$
$3S =$
$3S = \{1 + 2 + ... + n\}\{n + n + 1\}$
$S = \frac{n(n+1)(2n+1)}{6}$

$$\sum_{k=1}^{n}(-1)^{k-1}\left(\frac{1}{3}\right)^{k-1} = 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \frac{1}{243} + \frac{1}{729} - \cdots = \frac{3}{4}$$

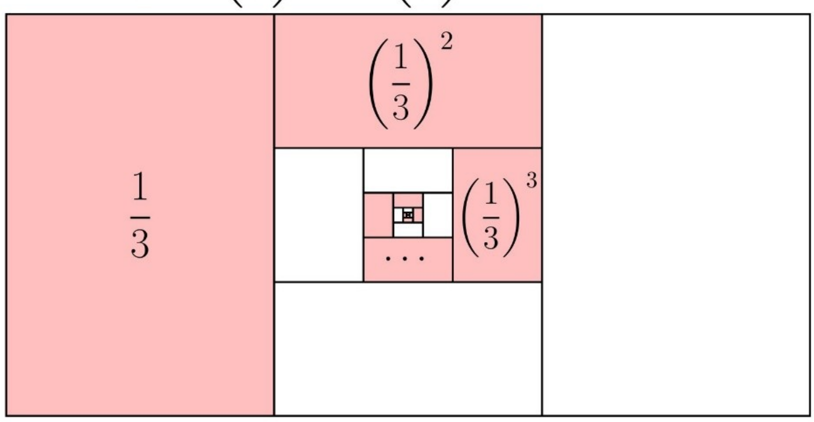




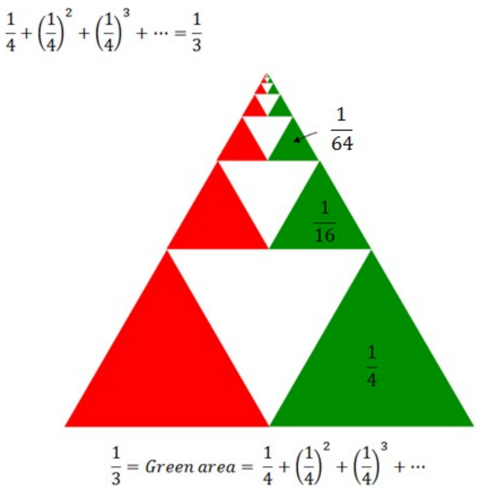


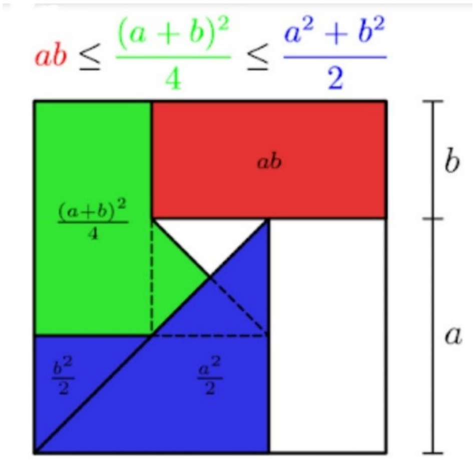
...nction φ where A) the transformed mean of a ...ibution E[φ(X)] (Jensen 1906) and B) where ... distributions than for the mean. (Merkle

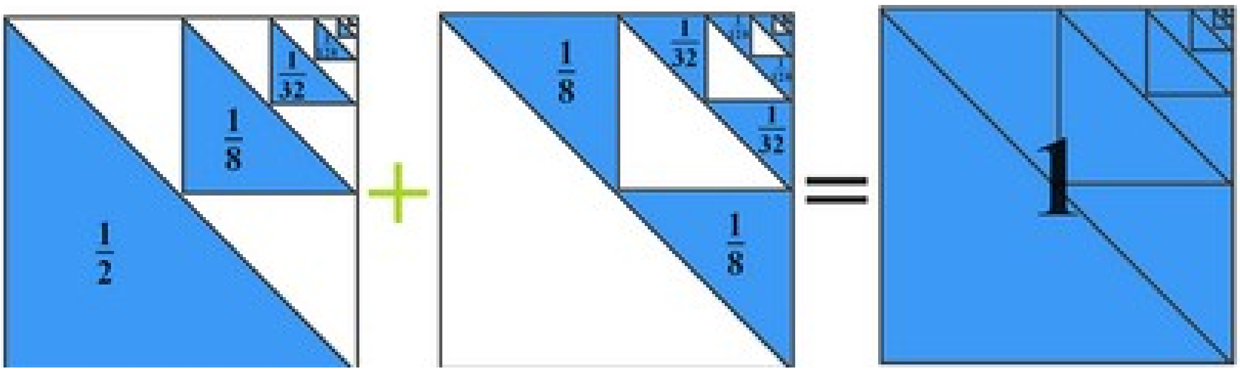
$$\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \cdots = \frac{1}{2}$$

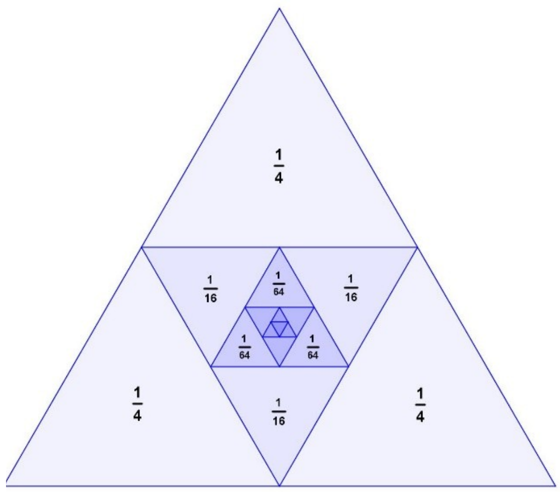


#ProofWithoutWords @TamasGorbe

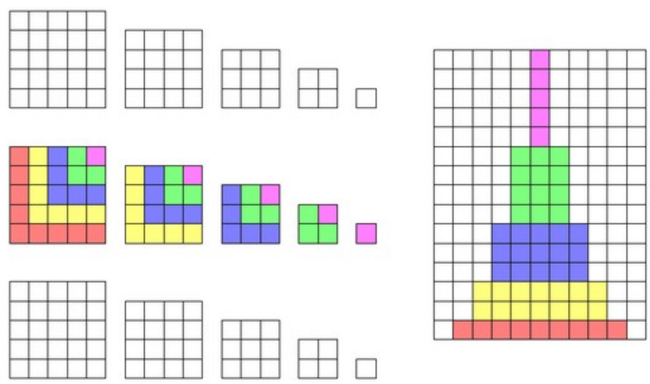








Here's a PWoW to show $1^2+2^2+3^2+\cdots+n^2=\dfrac{n(n+1)(2n+1)}{6}$.



100.39 An olympiad mathematical problem, proof without words and generalisation

The following problem occurred in a recent Spanish Mathematical Olympiad [1]:

Let a, b be two positive numbers with $a + b = 1$. Prove that

$$\left(1 + \frac{1}{a}\right)\left(1 + \frac{1}{b}\right) \geqslant 9.$$

Of course, the problem may be solved in several ways. We give below a proof using a diagram and no words.

Proof: We note that the given inequality may be rewritten as $(1 + a)(1 + b) \geqslant 9ab$. The truth of this inequality is apparent from the following diagram.

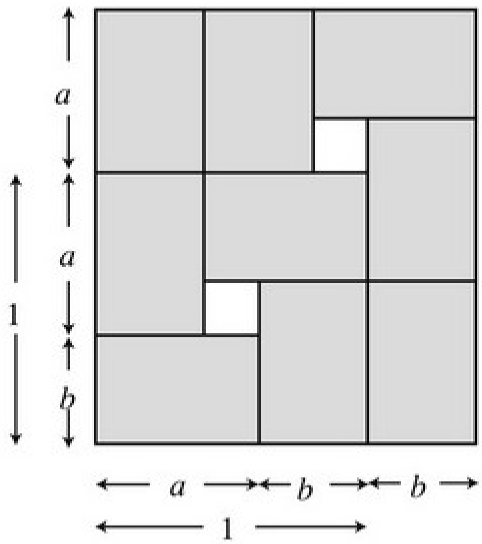


FIGURE 1

This proof is related to that given by Roger B. Nelsen [2], which also appears in [3, p. 62].

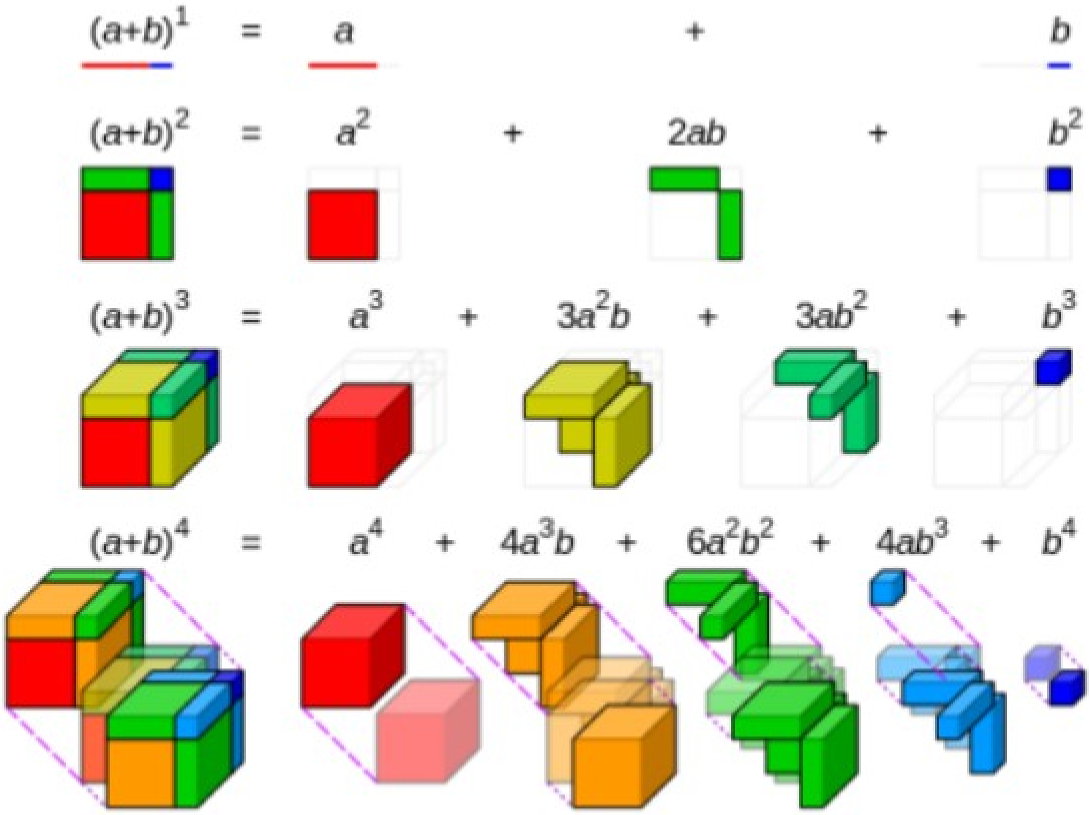
If we drop the condition that $a + b = 1$, Figure 1 may also be seen as a proof that $(2a + b)(2b + a) \geqslant 9ab$. Indeed it demonstrates that

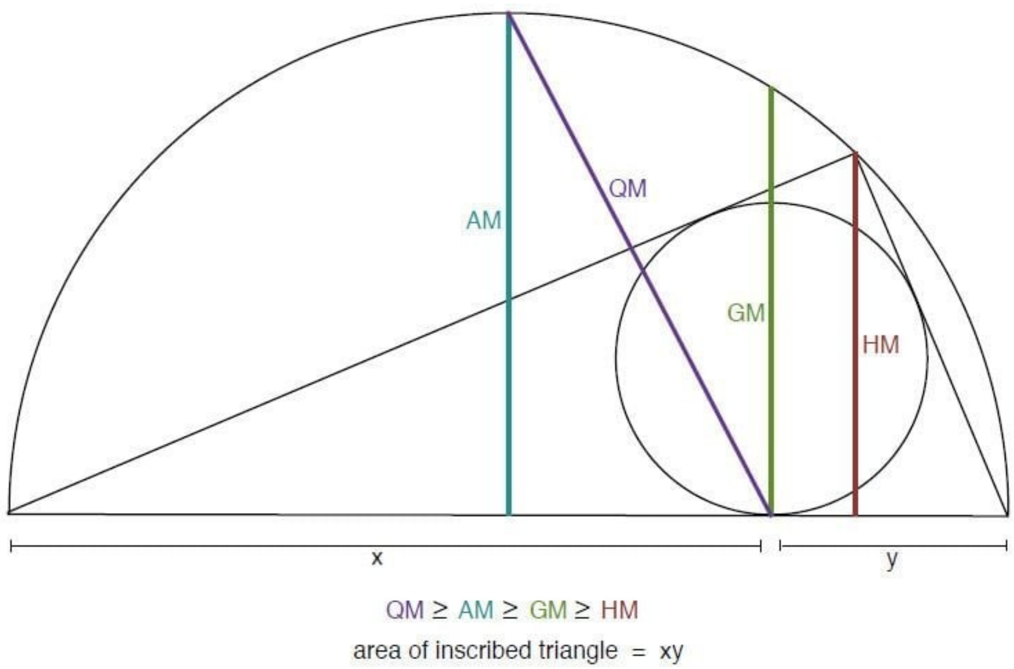
$$(2a + b)(2b + a) = 9ab + 2(a - b)^2.$$

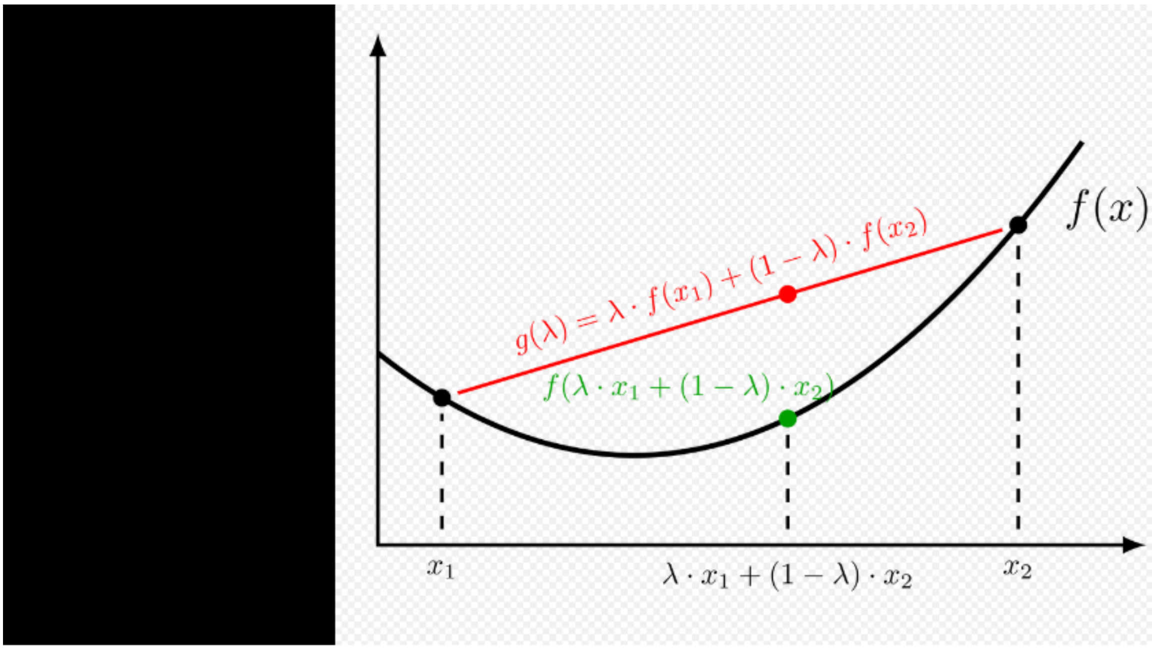
It is straightforward to check that this identity may be generalised as follows.

Theorem: Let a, b, m and n be positive real numbers. Then

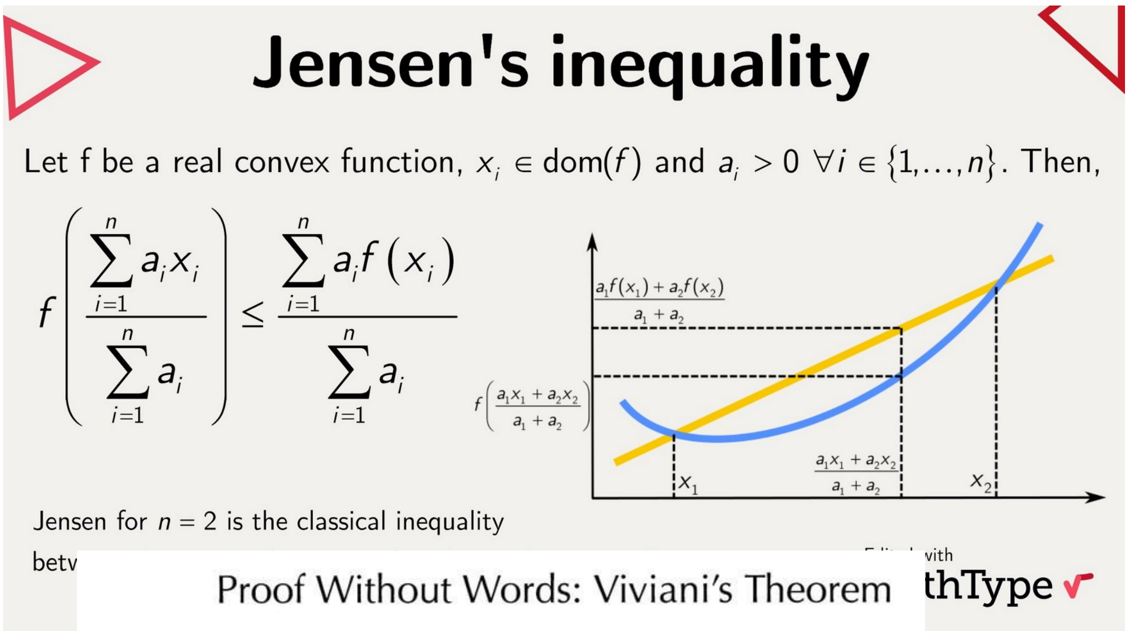
$$(ma + nb)(mb + na) = (m + n)^2 ab + mn(a - b)^2.$$


(a+b)1 = a + b
(a+b)2 = a2 + 2ab + b2
(a+b)3 = a3 + 3a2b + 3ab2 + b3
(a+b)4 = a4 + 4a3b + 6a2b2 + 4ab3 + b4


AM
QM
GM
HM
x
y
QM ≥ AM ≥ GM ≥ HM
area of inscribed triangle = xy

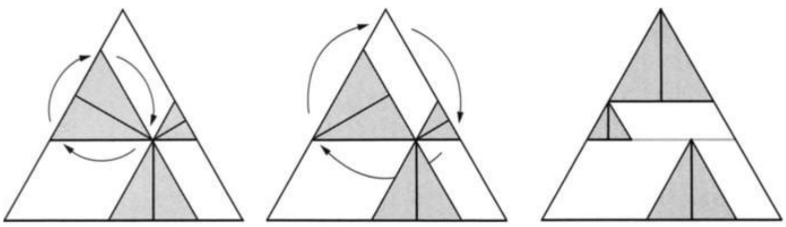


Η ανισότητα Γένσεν για την κυρτή συνάρτηση για δύο μεταβλητές , δίνει ότι για κάθε το κόκκινο σημείο είναι άνω του πράσινου.

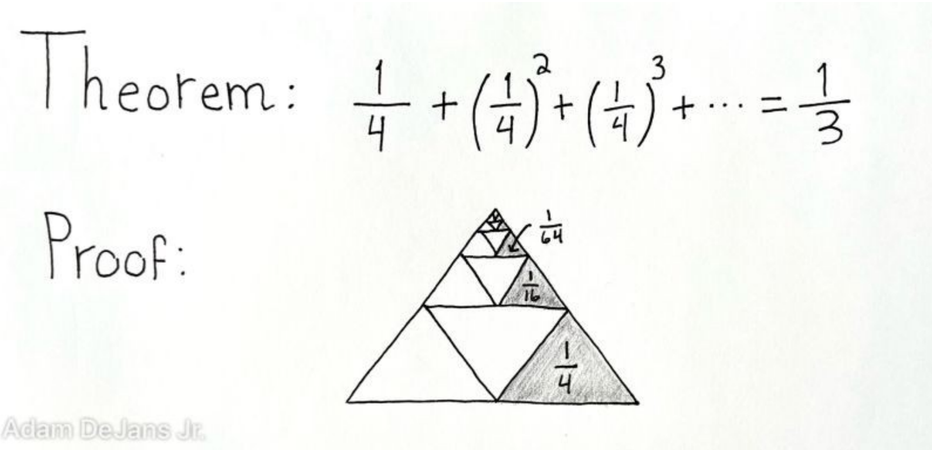


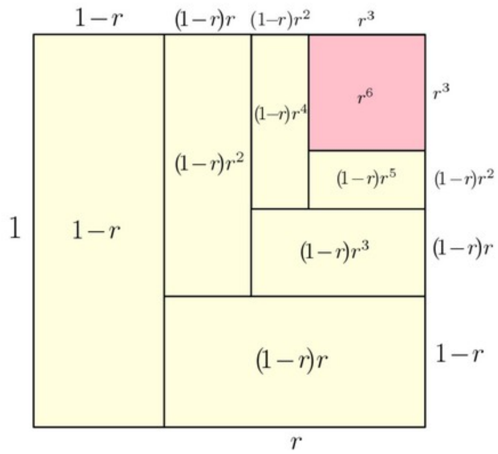
Proof Without Words: Viviani's Theorem

In an equilateral triangle, the sum of the distances from any interior point to the three sides is equal to the altitude of the triangle.

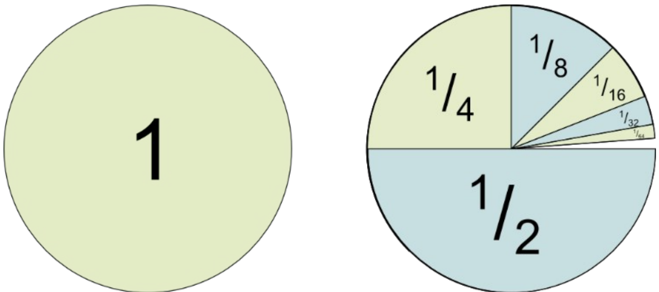


—KEN-ICHIROH KAWASAKI
NARA UNIVERSITY OF EDUCATION
TAKABATAKE-CHO, NARA
630–8528, JAPAN

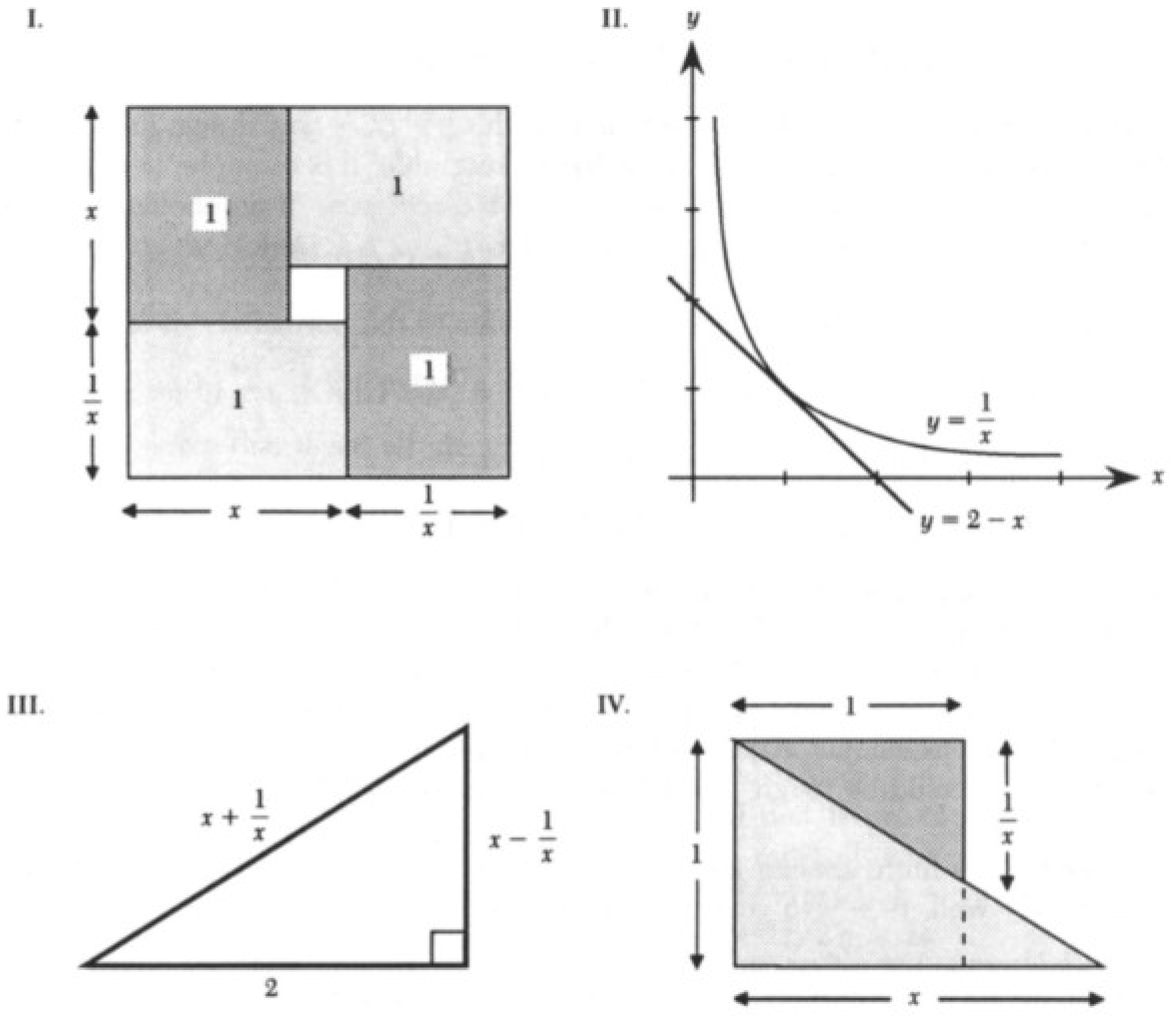




Proof of the finite sum of a geometric series, started at $k=0$ up to $k=n=5$. The area of the full square of unit side length is equal to the sum of the areas of all yellow rectangles plus the pink one, that is, $1=(1-r)\sum_{k=0}^{n} r^k + r^{n+1}$. Adapted from [1, p. 118].



Proof without Words: The Sum of a Positive Number and Its Reciprocal Is at Least Two (four proofs)



$$x \geq 1 \Rightarrow x + \frac{1}{x} \geq 2.$$

—Roger B. Nelsen
Lewis and Clark College
Portland, OR 97219

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

$$\frac{S}{2} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

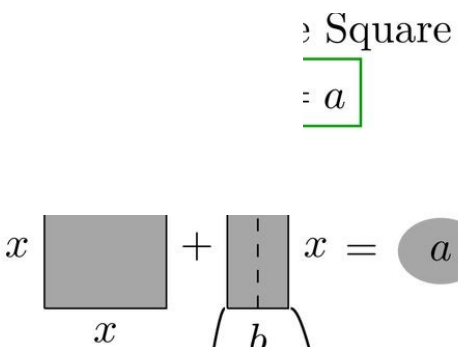
$$\frac{S}{2} = 1 + 0 + 0 + 0 + 0 + \cdots$$

$$\frac{S}{2} = 1$$

$$S = 2$$

$$\begin{aligned}
s_n &= ar^0 + ar^1 + \cdots + ar^{n-1}, \\
rs_n &= ar^1 + ar^2 + \cdots + ar^n, \\
s_n - rs_n &= ar^0 - ar^n, \\
s_n(1-r) &= a(1-r^n), \\
s_n &= a\left(\frac{1-r^n}{1-r}\right), \text{ for } r \neq 1.
\end{aligned}$$

$$S_n = a + \cancel{ar} + \cancel{ar^2} + \ldots + \cancel{ar^{n-1}}$$
$$-rS_n = \quad -\cancel{ar} - \cancel{ar^2} - \ldots - \cancel{ar^{n-1}} - ar^n$$
$$S_n - rS_n = a - ar^n$$
$$S_n(1-r) = a(1-r^n)$$
$$S_n = \frac{a(1-r^n)}{(1-r)}$$



Prove that there exist some irrational numbers x and y such that x^y is rational.

The proof follows like this:

Let $x = \sqrt{2}$

$y = \sqrt{2}$

now $z = x^y = \sqrt{2}^{\sqrt{2}}$

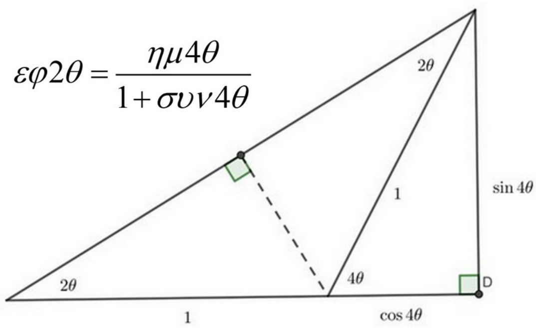
Case 1: If z is rational. Then we have successfully completed the proof.

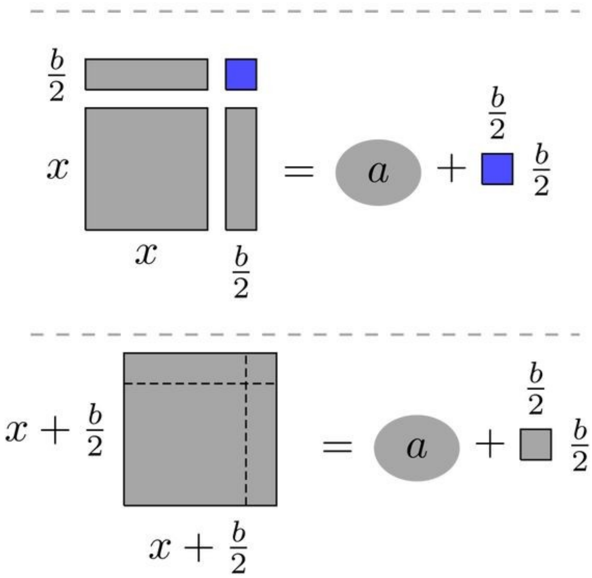
Case 2: If z is irrational:

Then choose x = z and now:

$$res = x^y = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^2 = 2$$

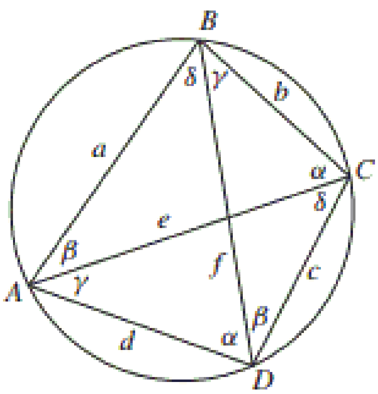
$$\varepsilon\varphi 2\theta = \frac{\eta\mu 4\theta}{1+\sigma\upsilon\nu 4\theta}$$



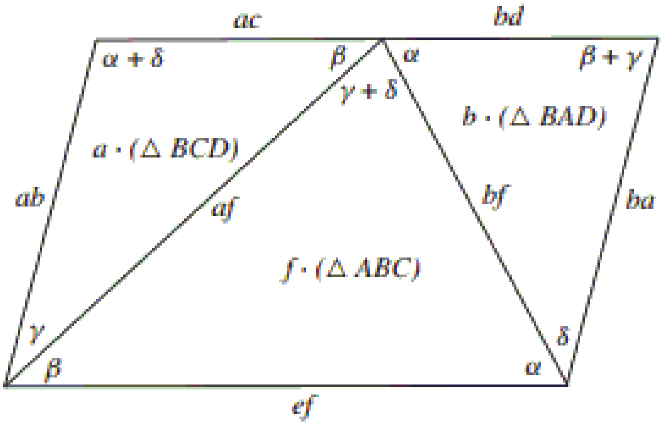


$$\left(x + \tfrac{b}{2}\right)^2 = a + \left(\tfrac{b}{2}\right)^2$$

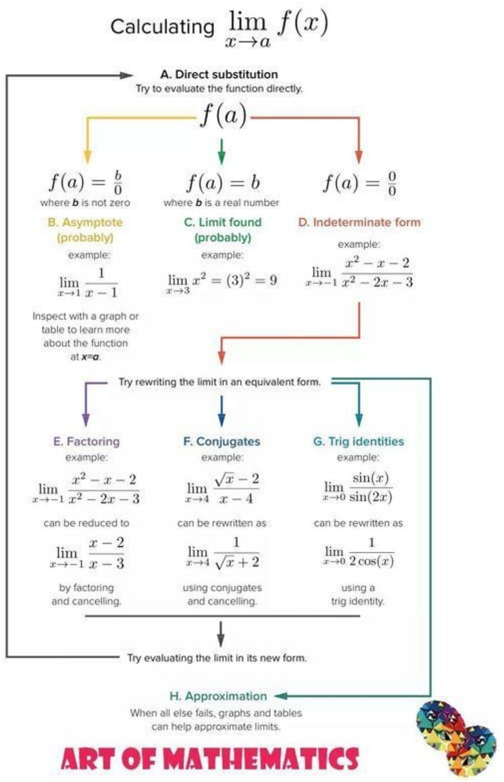
$$\boxed{x = -\tfrac{b}{2} \pm \sqrt{a + \tfrac{b^2}{4}}}$$

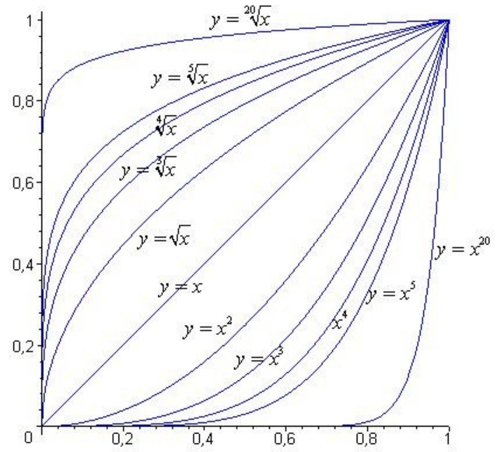


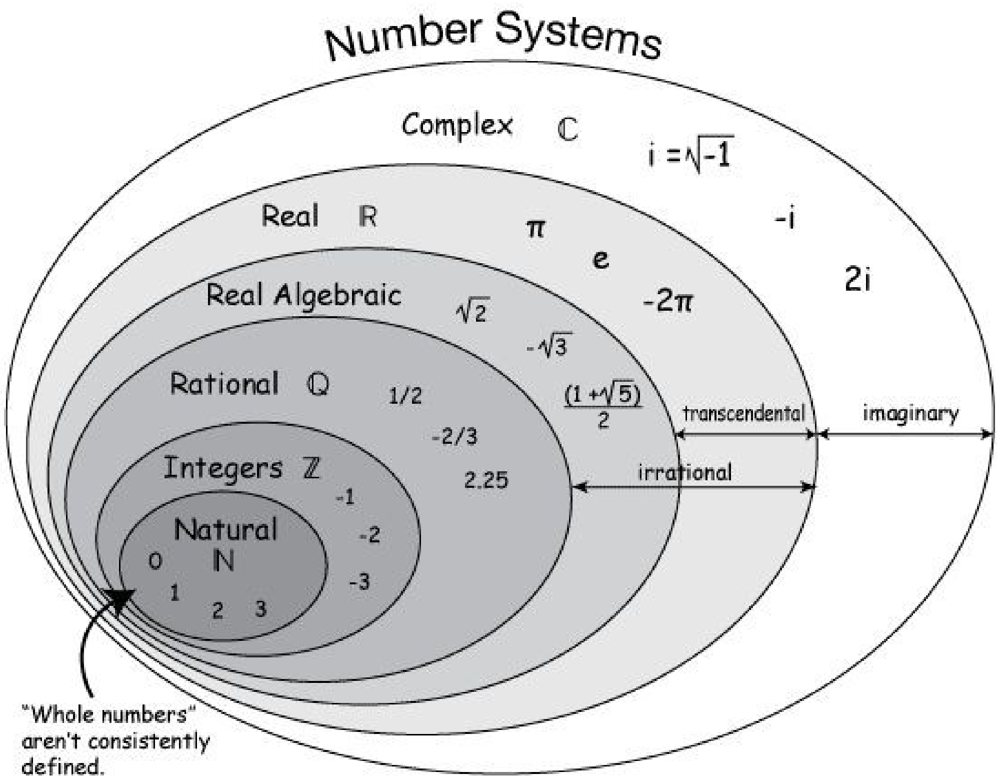
$\alpha + \beta + \gamma + \delta = 180°$

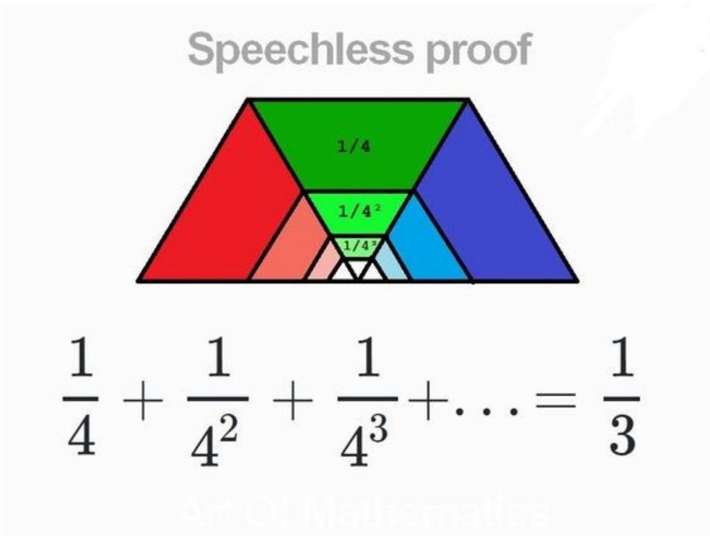


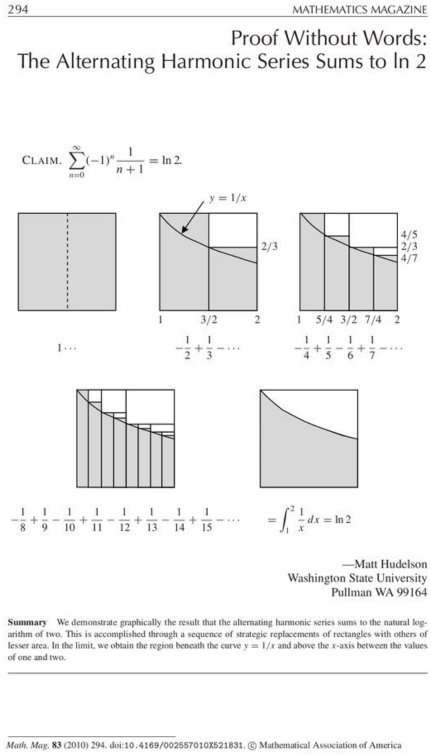
$\therefore\ ef = ac + bd$


Calculating $\lim_{x \to a} f(x)$
A. Direct substitution
Try to evaluate the function directly.
$f(a)$
$f(a) = \frac{b}{0}$
where b is not zero
B. Asymptote (probably)
example:
$\lim_{x \to 1} \frac{1}{x-1}$
Inspect with a graph or table to learn more about the function at x=a.
$f(a) = b$
where b is a real number
C. Limit found (probably)
example:
$\lim_{x \to 3} x^2 = (3)^2 = 9$
$f(a) = \frac{0}{0}$
D. Indeterminate form
example:
$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 2x - 3}$
Try rewriting the limit in an equivalent form.
E. Factoring
example:
$\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 2x - 3}$
can be reduced to
$\lim_{x \to -1} \frac{x-2}{x-3}$
by factoring and cancelling.
F. Conjugates
example:
$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$
can be rewritten as
$\lim_{x \to 4} \frac{1}{\sqrt{x} + 2}$
using conjugates and cancelling.
G. Trig identities
example:
$\lim_{x \to 0} \frac{\sin(x)}{\sin(2x)}$
can be rewritten as
$\lim_{x \to 0} \frac{1}{2\cos(x)}$
using a trig identity.
Try evaluating the limit in its new form.
H. Approximation
When all else fails, graphs and tables can help approximate limits.
ART OF MATHEMATICS


$y = \sqrt[20]{x}$
$y = \sqrt[5]{x}$
$\sqrt[4]{x}$
$y = \sqrt[3]{x}$
$y = \sqrt{x}$
$y = x$
$y = x^2$
$y = x^3$
x^4
$y = x^5$
$y = x^{20}$
0
0,2
0,4
0,6
0,8
1


Number Systems
Complex $\mathbb{C}$
$i = \sqrt{-1}$
-i
2i
Real $\mathbb{R}$
π
e
-2π
Real Algebraic
$\sqrt{2}$
$-\sqrt{3}$
$\frac{(1+\sqrt{5})}{2}$
Rational $\mathbb{Q}$
1/2
-2/3
2.25
Integers $\mathbb{Z}$
-1
-2
-3
Natural $\mathbb{N}$
0
1
2
3
transcendental
imaginary
irrational
"Whole numbers" aren't consistently defined.




294 MATHEMATICS MAGAZINE

Proof Without Words:
The Alternating Harmonic Series Sums to ln 2

CLAIM. $\sum_{n=0}^{\infty}(-1)^n\frac{1}{n+1}=\ln 2.$

$y = 1/x$

2/3

4/5
2/3
4/7

1 3/2 2

1 5/4 3/2 7/4 2

$1\cdots$

$-\frac{1}{2}+\frac{1}{3}-\cdots$

$-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\cdots$

$-\frac{1}{8}+\frac{1}{9}-\frac{1}{10}+\frac{1}{11}-\frac{1}{12}+\frac{1}{13}-\frac{1}{14}+\frac{1}{15}-\cdots$

$=\int_1^2\frac{1}{x}\,dx=\ln 2$

—Matt Hudelson
Washington State University
Pullman WA 99164

Summary We demonstrate graphically the result that the alternating harmonic series sums to the natural logarithm of two. This is accomplished through a sequence of strategic replacements of rectangles with others of lesser area. In the limit, we obtain the region beneath the curve $y = 1/x$ and above the x-axis between the values of one and two.

Math. Mag. **83** (2010) 294. doi:10.4169/002557010X521831. © Mathematical Association of America

Probability

Multiplication Rule

Independent Events

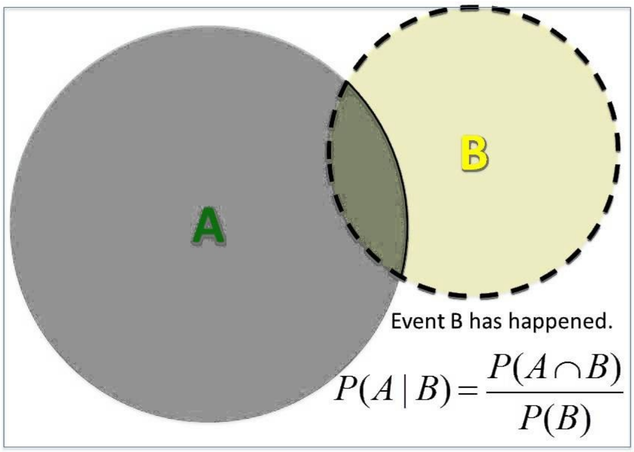
$$P(X \cap Y) = P(X) \bullet P(Y)$$

Dependent Events

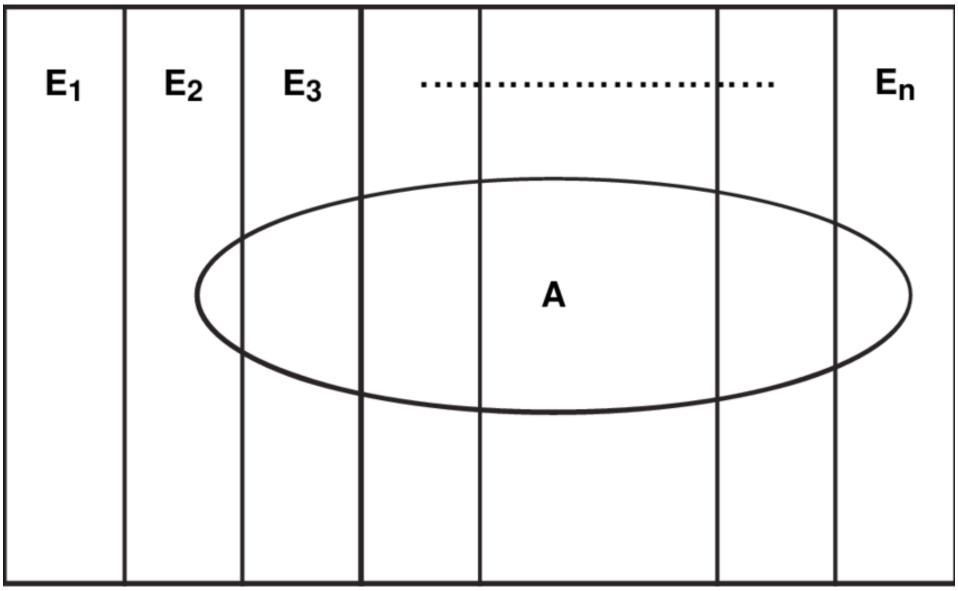
$$P(X \cap Y) = P(Y) \bullet P(X \mid Y)$$

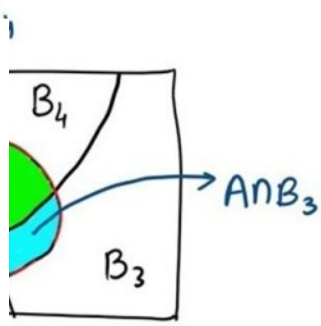
Bayes' Theorem

$$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$$



Given that B has happened, the probability that A will happen, too, is just the area ratio of the banana-shaped region to the B circle.





Derivation of Bayes' Theorem

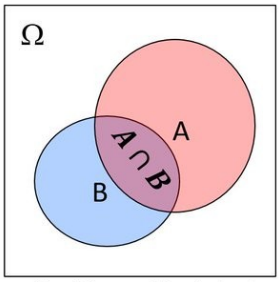
Michael Pyrcz, University of Texas at Austin (Twitter @GeostatsGuy)

Bayes' Theorem is central to Bayesian Statistics. It allows for: (1) the updating prior probability distribution with a likelihood function based on new information, and (2) the calculation of the conditional probability, P(A|B), given another calculated conditional probability, P(B|A). Did you know that is it derived from basic probability logic?

Rule of Multiplication:

$$P(B \cap A) = P(A|B)\,P(B)$$

$$P(A \cap B) = P(B|A)\,P(A)$$



Venn Diagram - Marginal and joint probability

It follows that:

$$P(B \cap A) = P(A \cap B)$$

Therefore when we substitute we get:

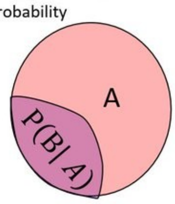
$$P(A|B)\,P(B) = P(B|A)\,P(A)$$

Now we have Bayes' Theorem!

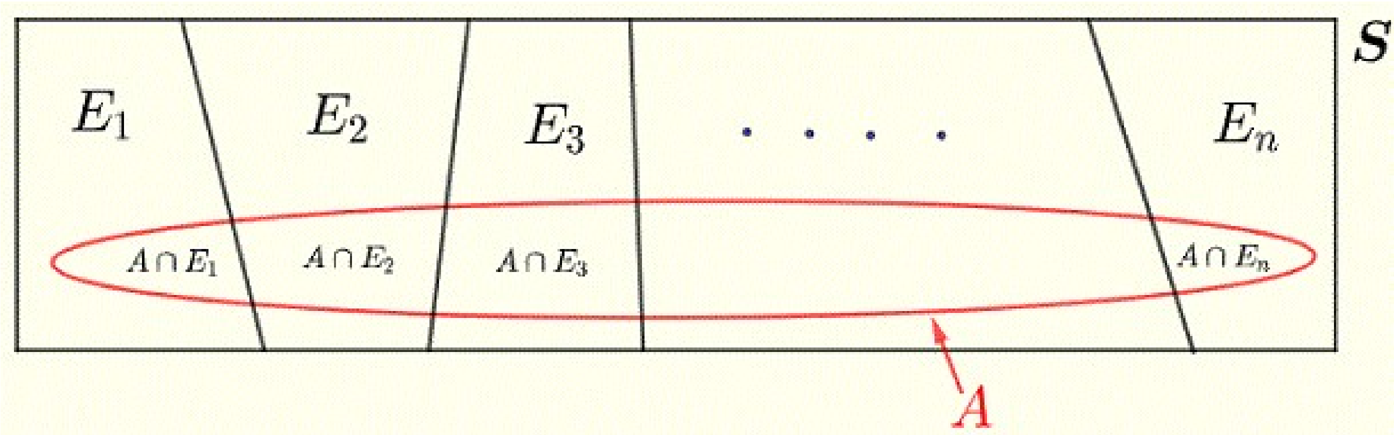
$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

The terms are known as:

$$\text{Posterior} = \frac{\text{Likelihood x Prior}}{\text{Evidence}}$$



Venn Diagram - Marginal and conditional probability



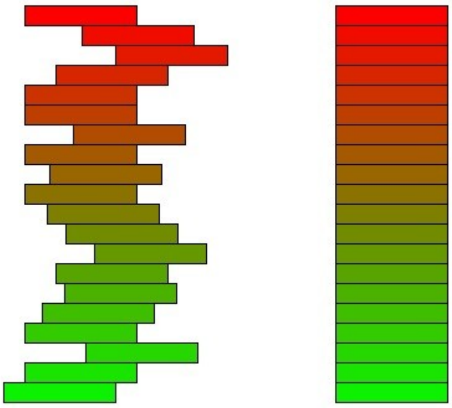
$$P(A)P(E_i|A) = P(E_i)P(A|E_i)$$

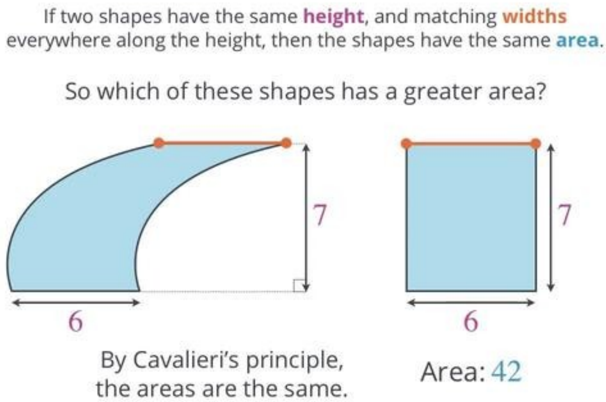
which gives

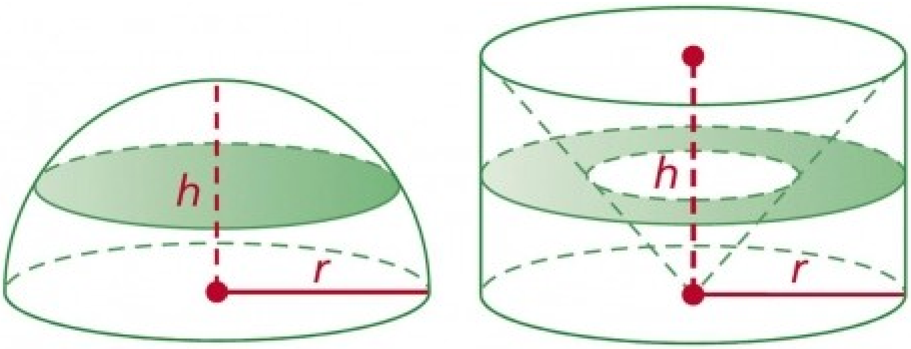
$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{P(A)}$$

Substitute $P(A)$ by the above sum to write Bayes' theorem as follows

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{i=1}^{n} P(A|E_i)P(E_i)}$$

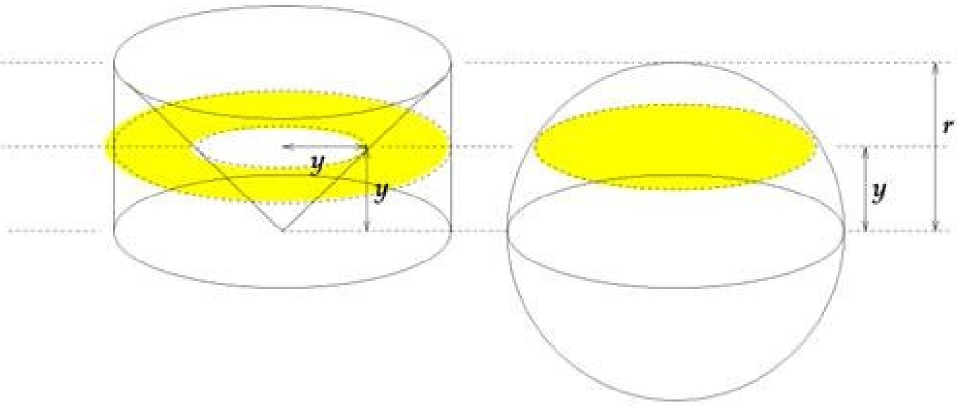



If two shapes have the same height, and matching widths
everywhere along the height, then the shapes have the same area.
So which of these shapes has a greater area?
7
6
7
6
By Cavalieri's principle,
the areas are the same.
Area: 42

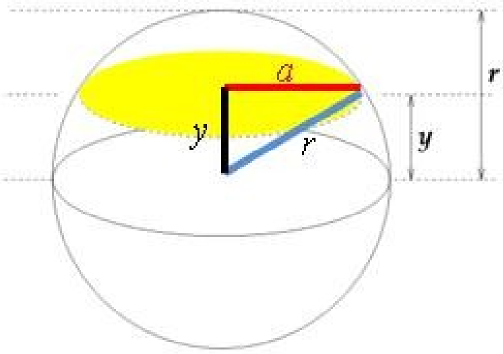

h
r
h
r

Proof of volume of sphere using Cavalieri's Principle

Shared by: Muhammad Na'im Bin Sa'dollah



Aim: To prove that at any height of y, the area of the doughnut (in the cylinder) is equal to the area of the circle (in the sphere)



Looking at the sphere,

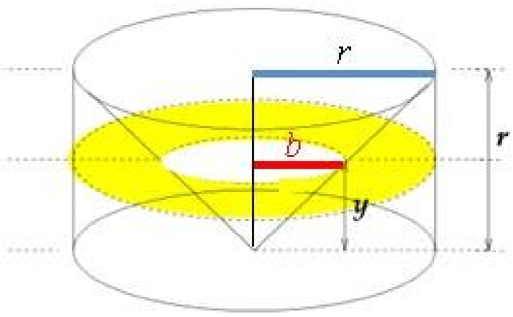
Let the radius of the circle be a.

By Pythagoras' Theorem,

$$a = \sqrt{r^2 - y^2}$$

Therefore,

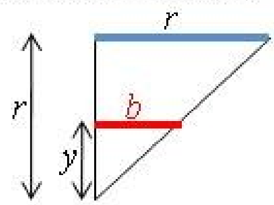
Area of the circle = $\pi(r^2 - y^2)$



Looking at the cylinder,

Let the radius of the inner circle be b.

By Similar Triangles,



$$\frac{\text{Base of smaller } \Delta}{\text{Base of larger } \Delta} = \frac{\text{Height of smaller } \Delta}{\text{Height of larger } \Delta}$$

$$\frac{b}{r} = \frac{y}{r}$$

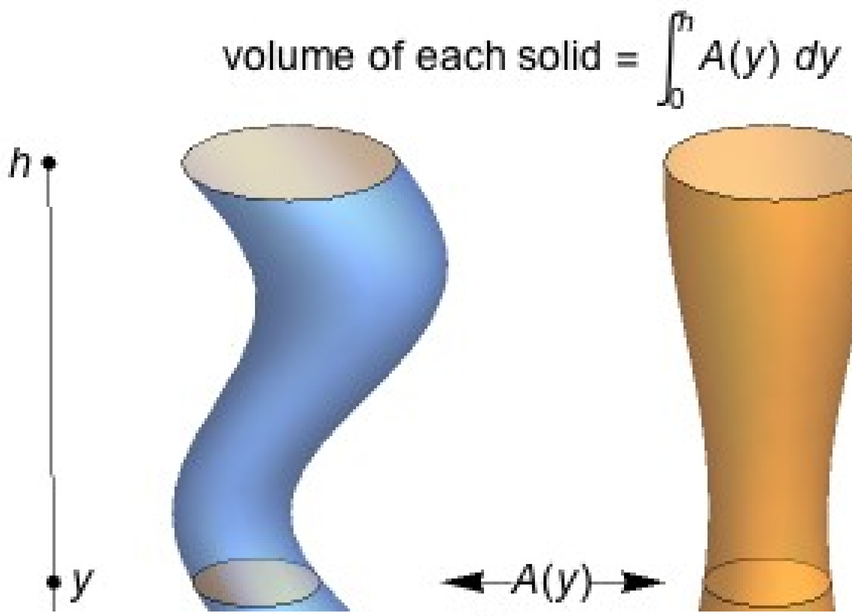
Therefore, $b = y$

Area of doughnut = $\pi r^2 - \pi y^2$

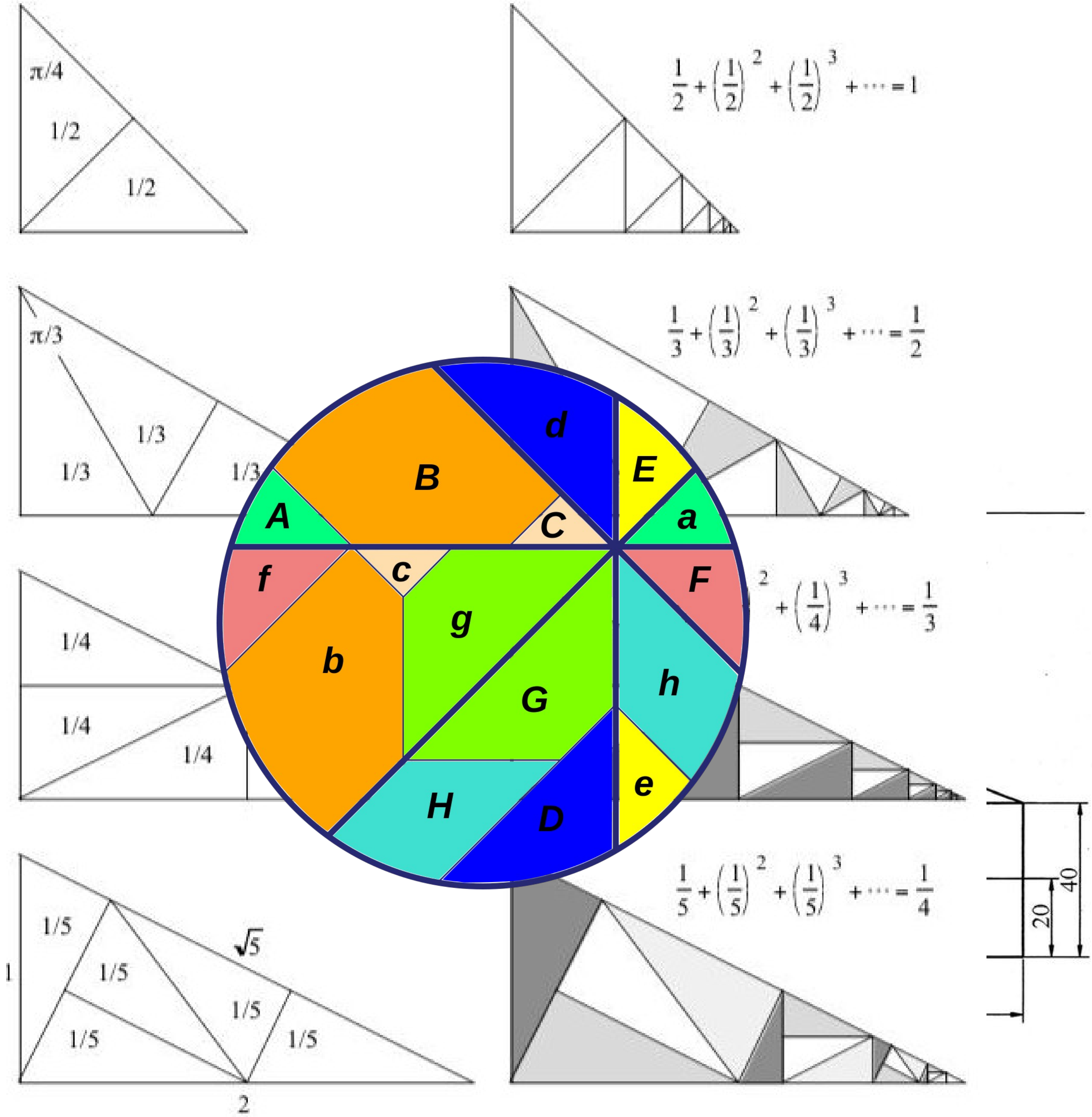
$= \pi(r^2 - y^2)$

Therefore,

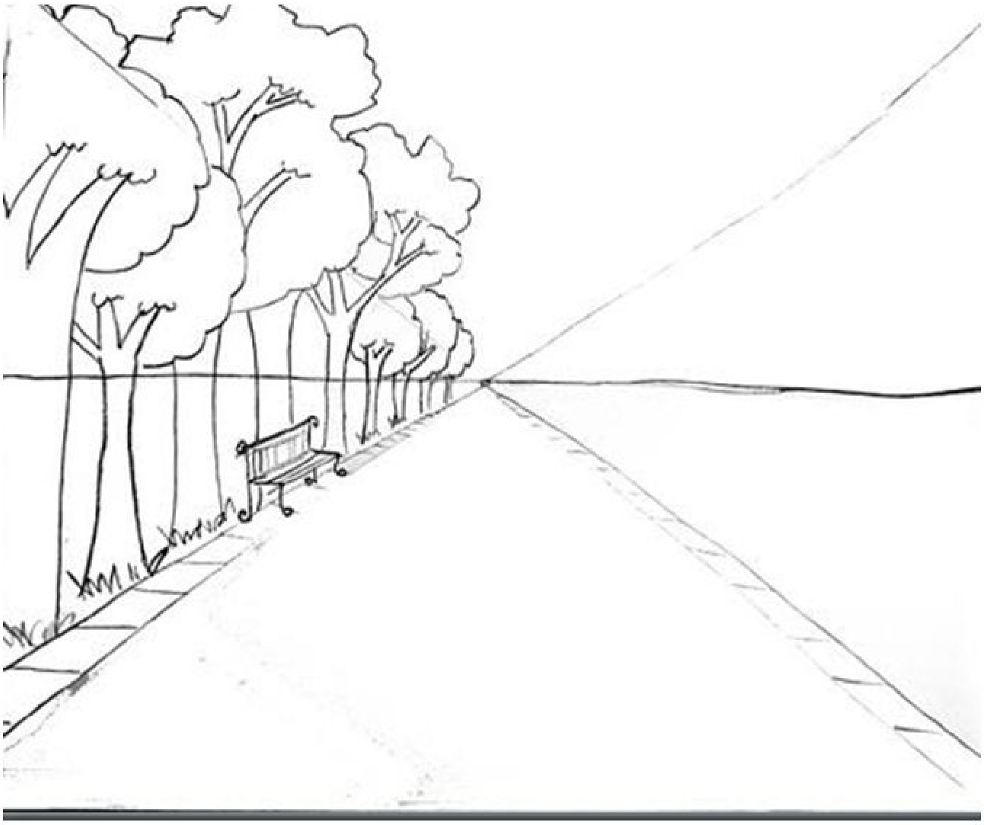
At any height y, Area of doughnut = Area of circle (shown)



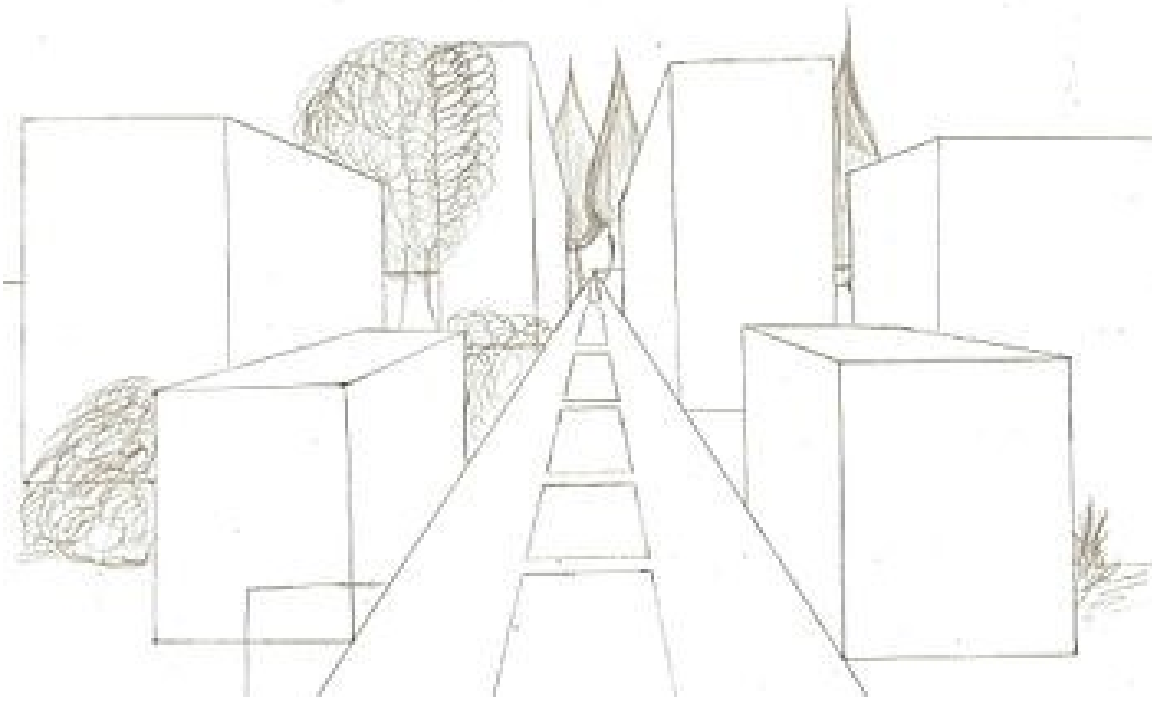
volume of each solid = $\int_0^h A(y)\,dy$
h
y
A(y)

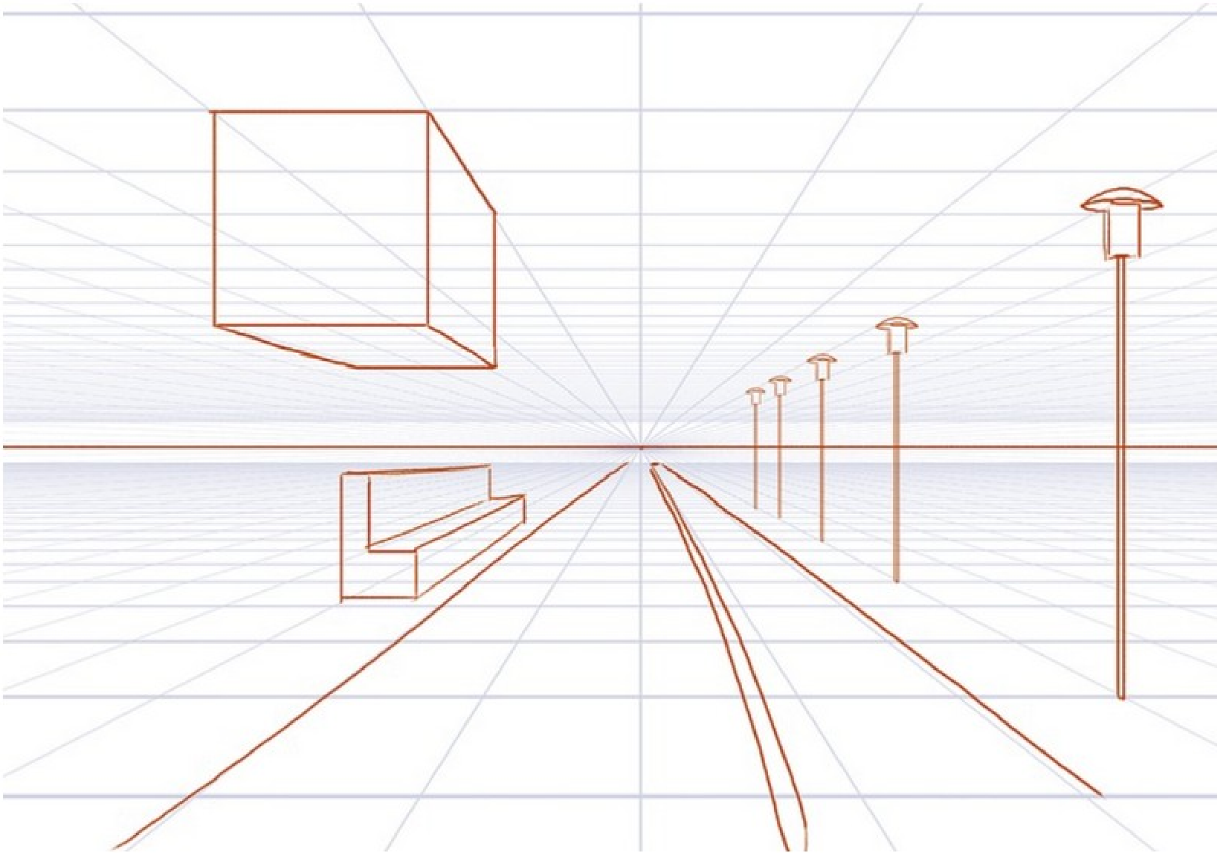


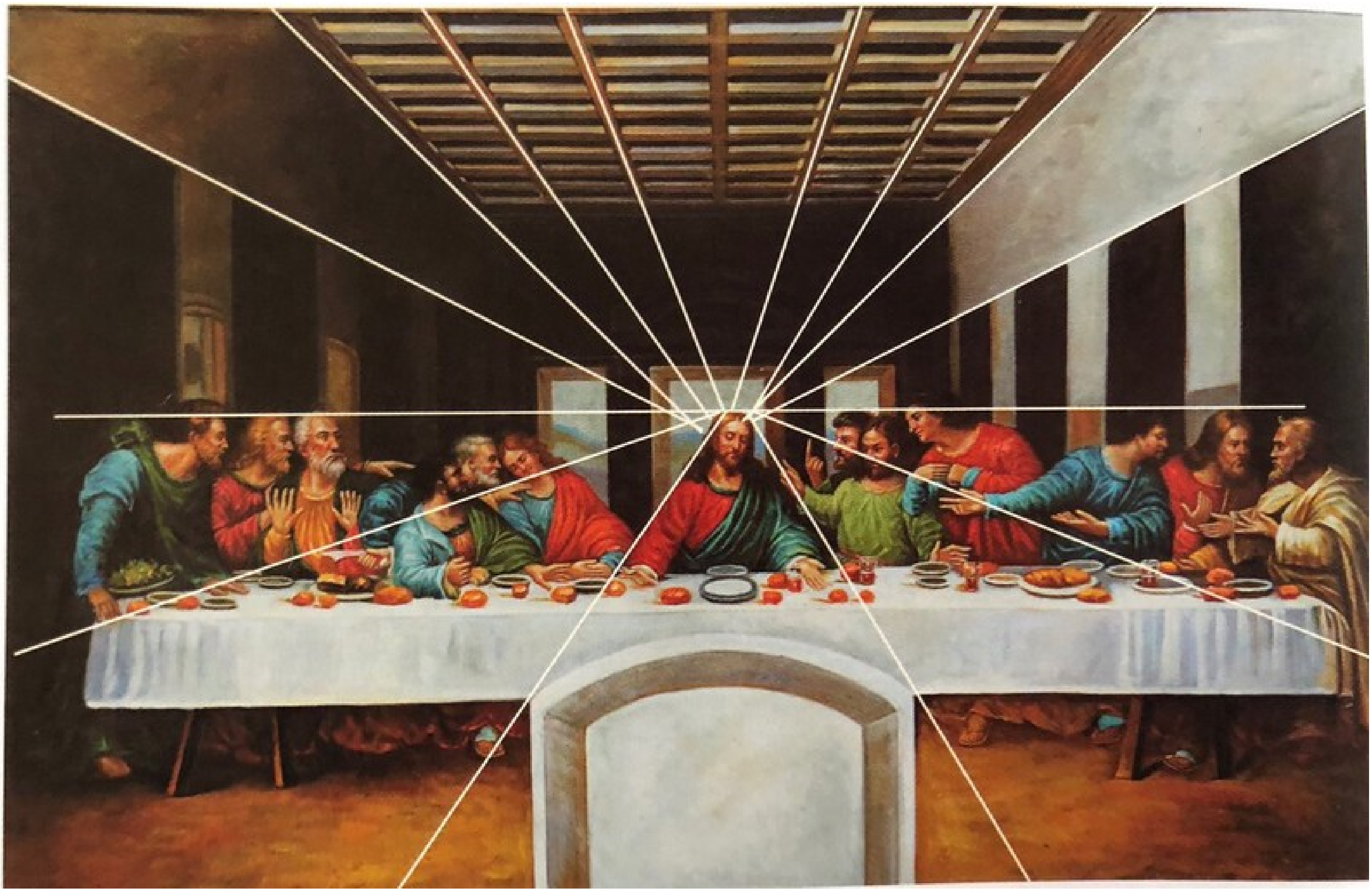
π/4
1/2
1/2
$\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\cdots=1$
π/3
1/3
1/3
1/3
$\frac{1}{3}+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3+\cdots=\frac{1}{2}$
1/4
1/4
1/4
$+\left(\frac{1}{4}\right)^3+\cdots=\frac{1}{3}$
d
B
E
A
C
a
f
c
F
g
b
h
G
H
D
e
20
40
1/5
1/5
1/5
1/5
1/5
$\sqrt{5}$
1
2
$\frac{1}{5}+\left(\frac{1}{5}\right)^2+\left(\frac{1}{5}\right)^3+\cdots=\frac{1}{4}$

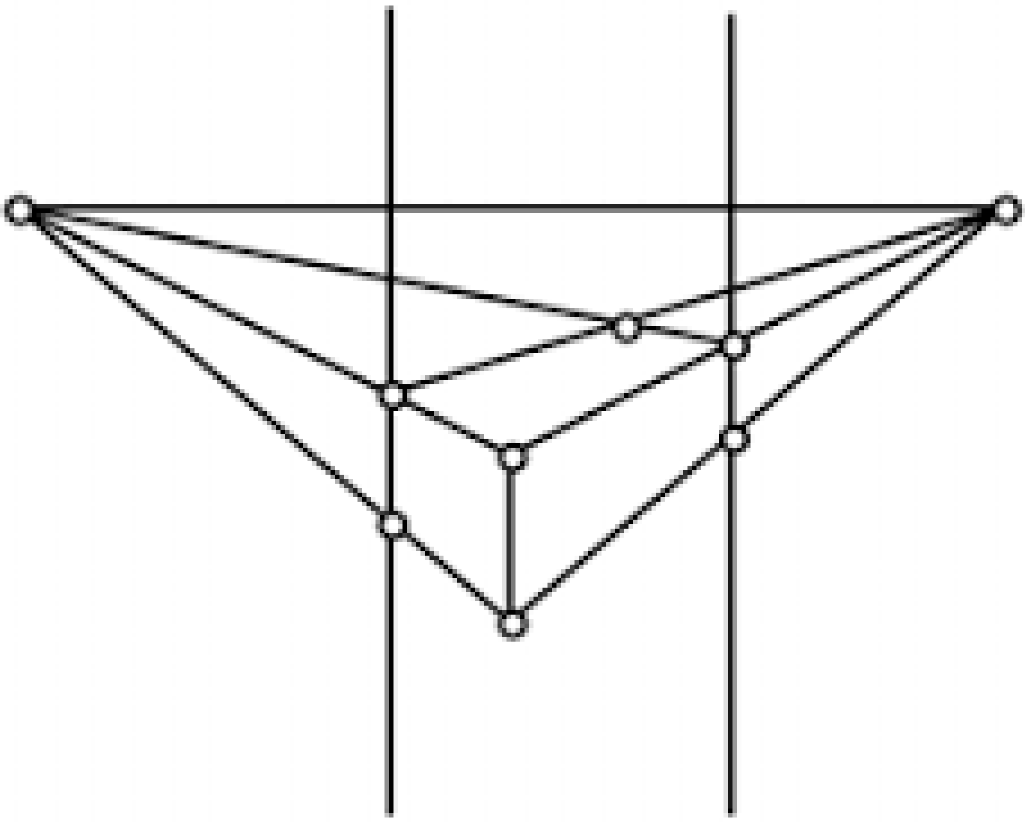


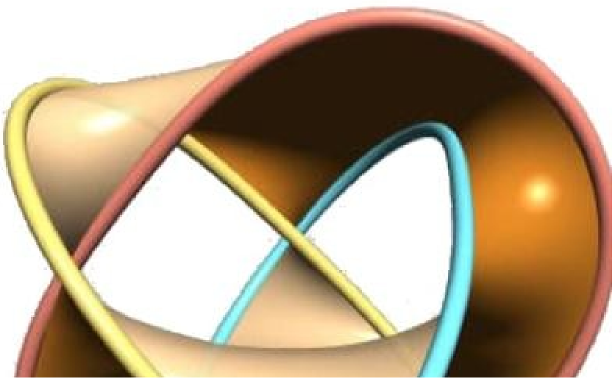
Προοπτική με ένα σημείο φυγής

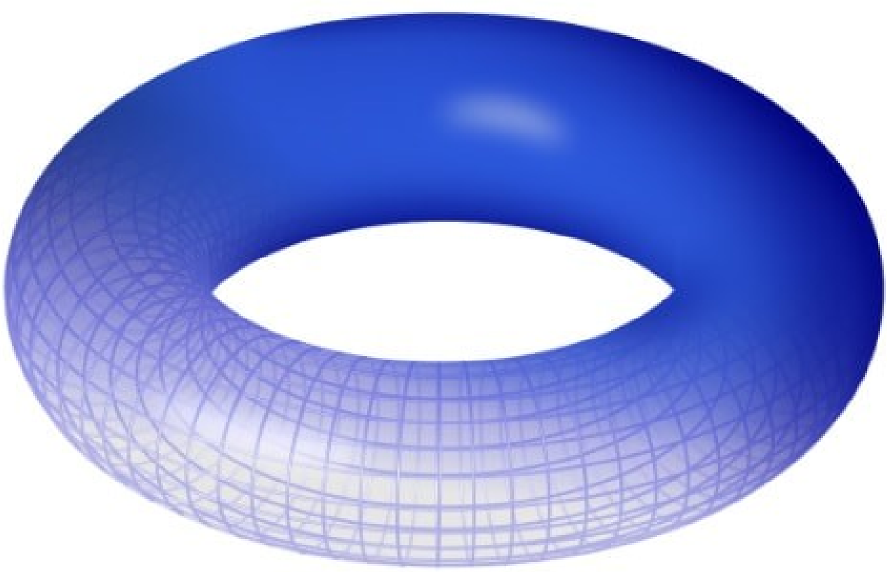


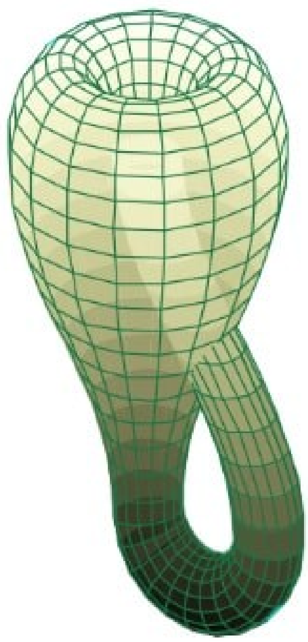


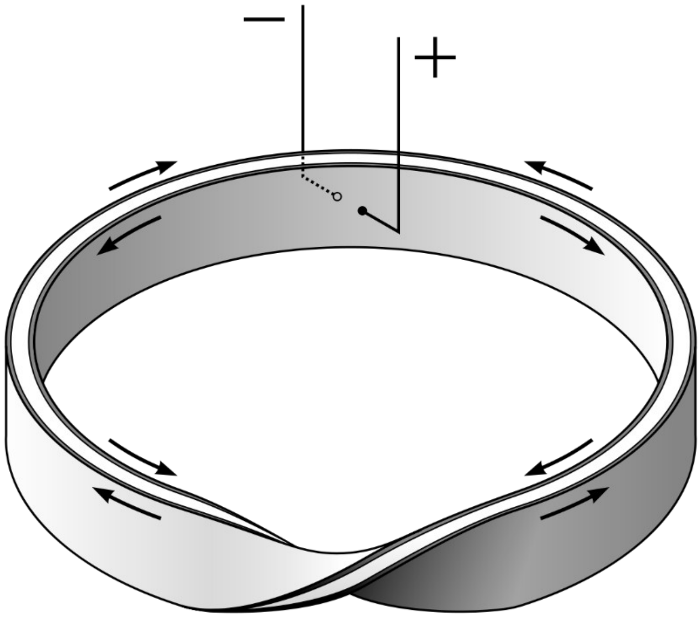


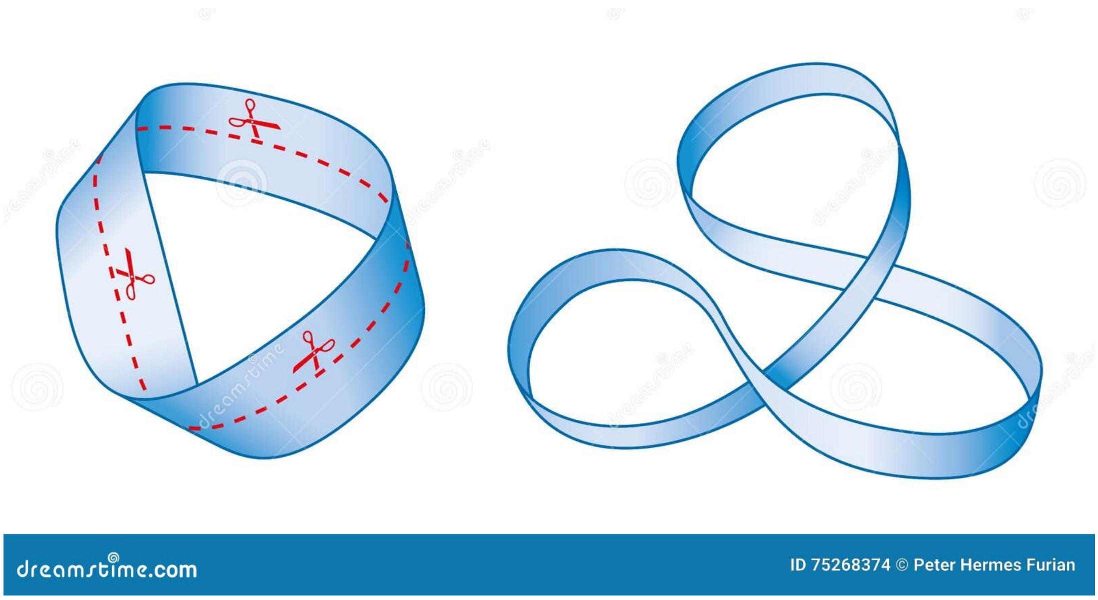


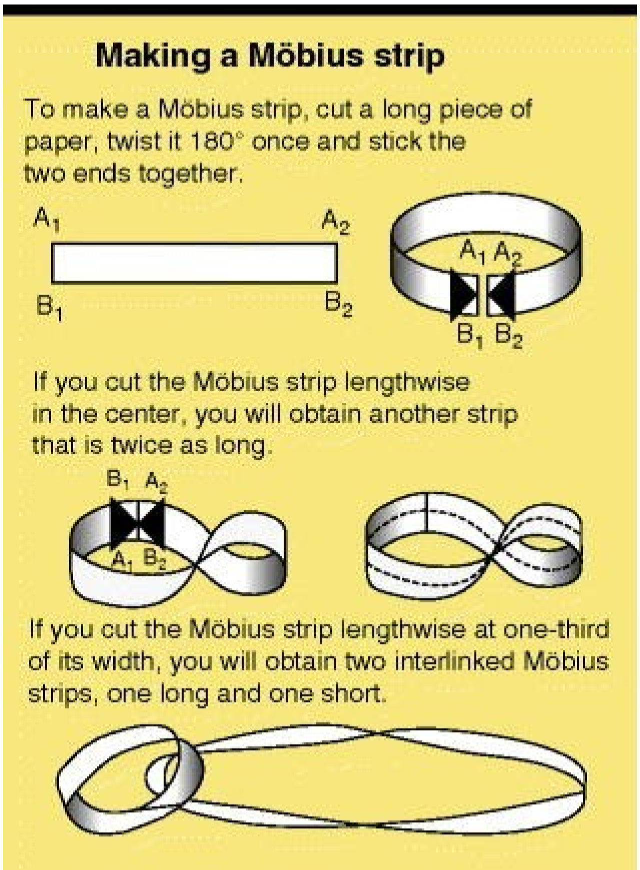






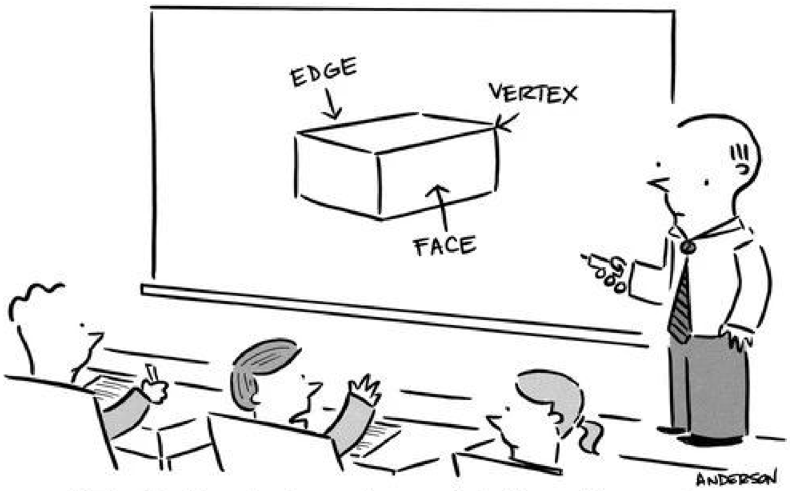

−
+


dreamstime.com
ID 75268374 © Peter Hermes Furian

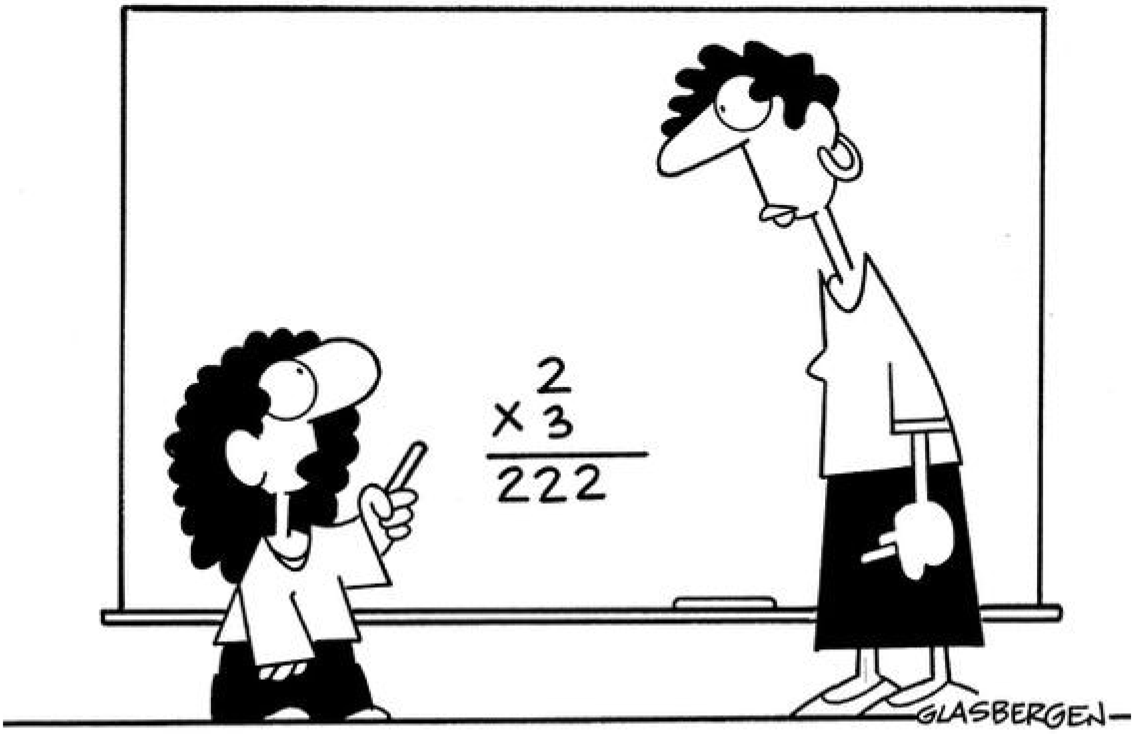


Newspaper Expresso

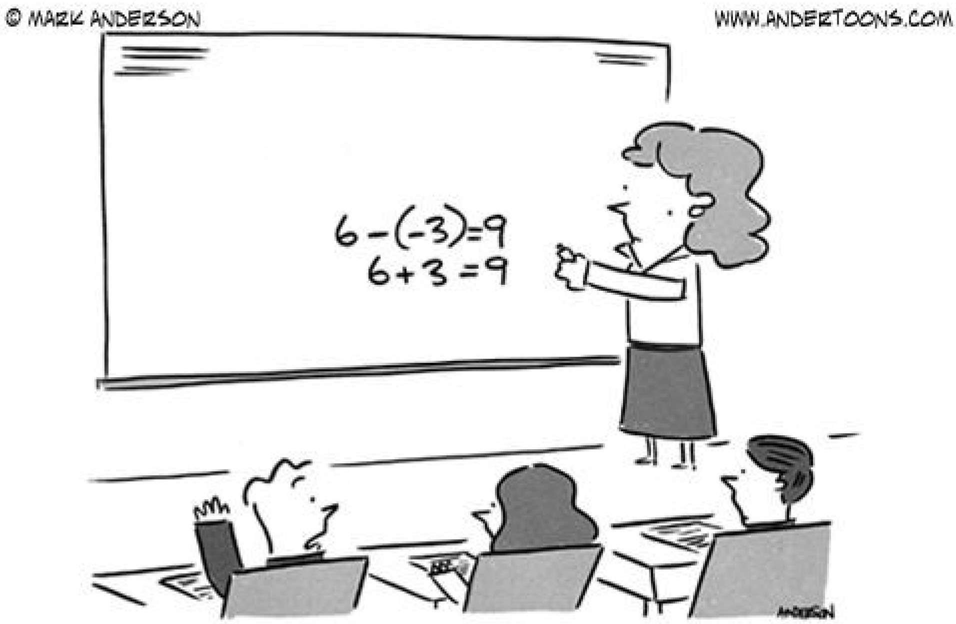
"What about Instagram?"



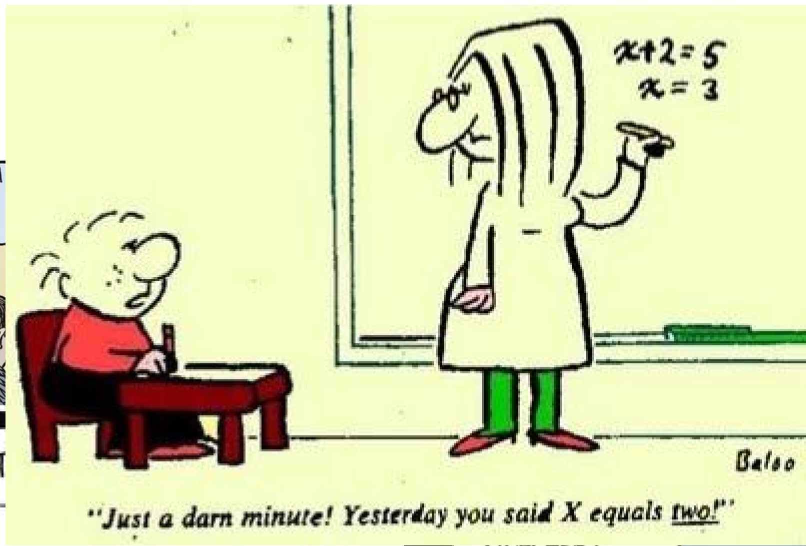
"That's fine, but you haven't told us the most important part - what's *in* it?!"



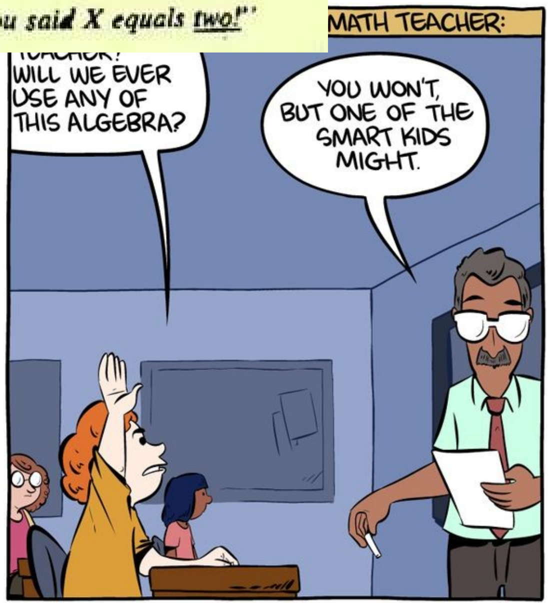
"What do you mean, it's the wrong kind of right?"

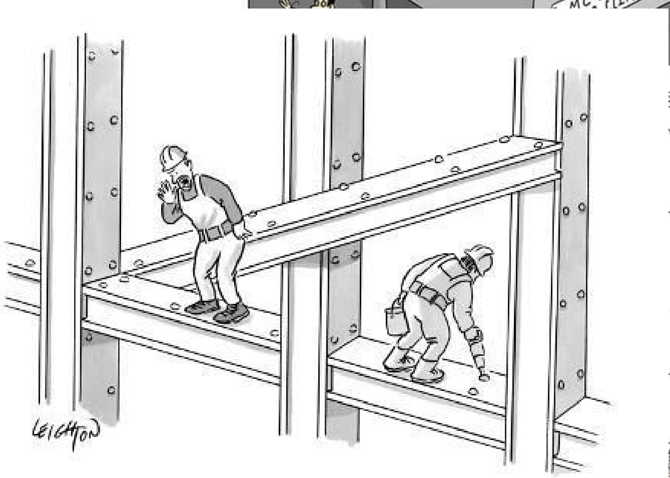


"So in English a double negative is bad, but in math it's *a positive*?"



"Just a darn minute! Yesterday you said X equals two!"

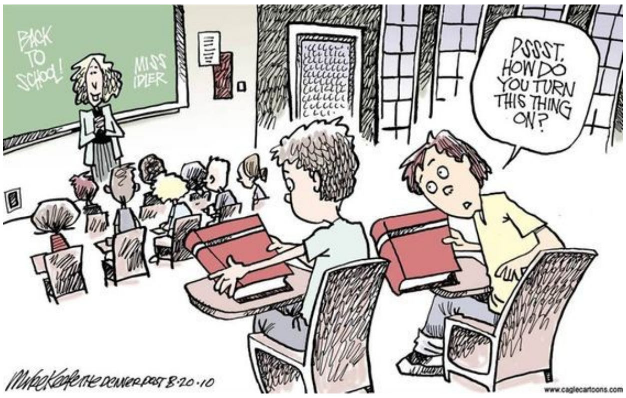


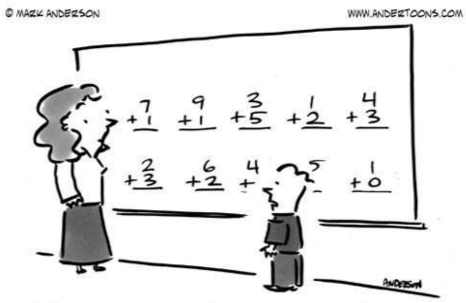


"Escher! Get your ass up here."

M. C. Escher as a Child

"Who is putting all the Math books in the Horror section?"

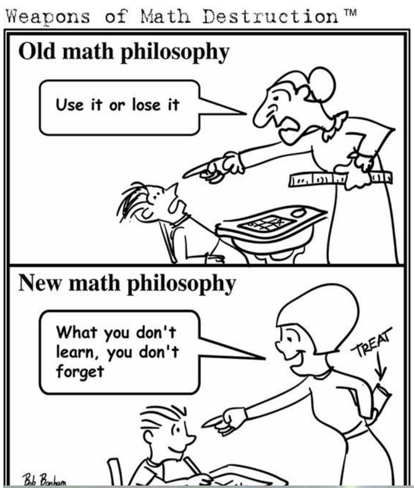


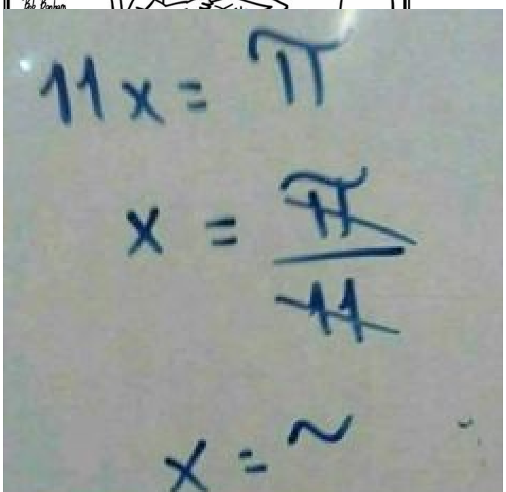


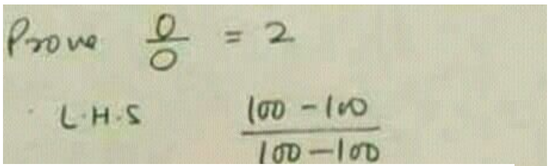
"All I'm saying is we plug these into Excel, let it do its thing, and then we can all play until lunch!"

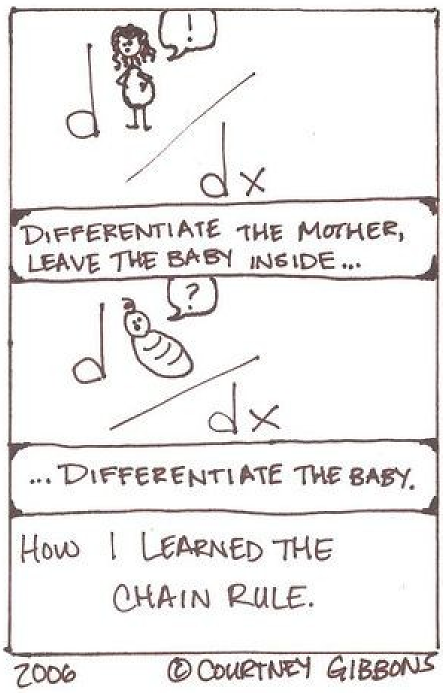
Can You Find The Mistake?

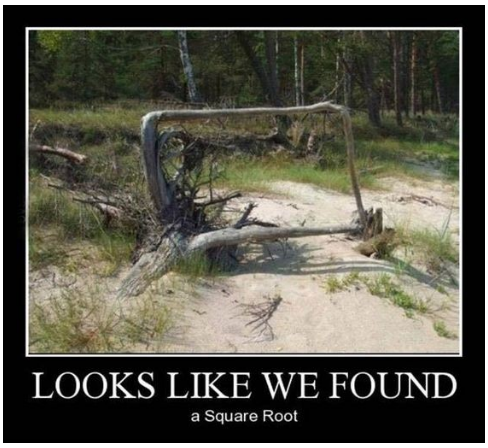
$$9 - 24 = 25 - 40$$
$$9 - 24 + 16 = 25 - 40 + 16$$
$$(3 - 4)^2 = (5 - 4)^2$$
$$3 - 4 = 5 - 4$$
$$\boxed{3 = 5}$$

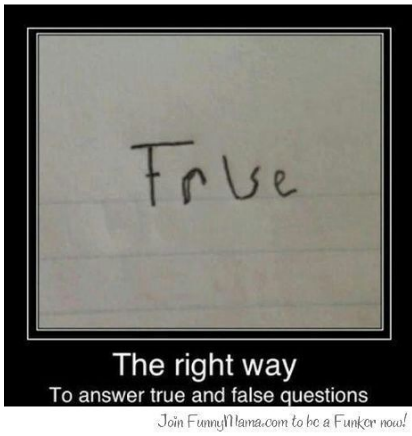


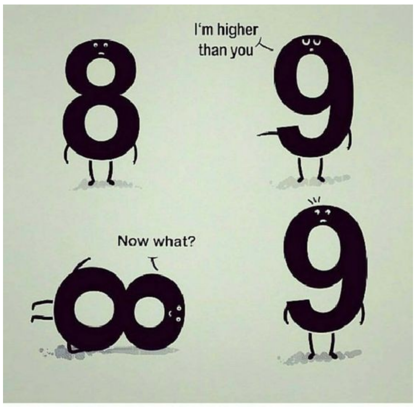








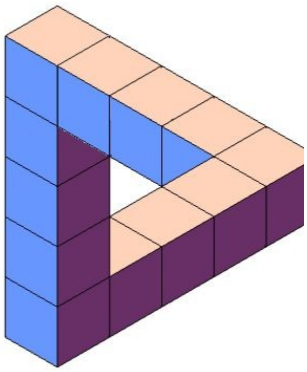


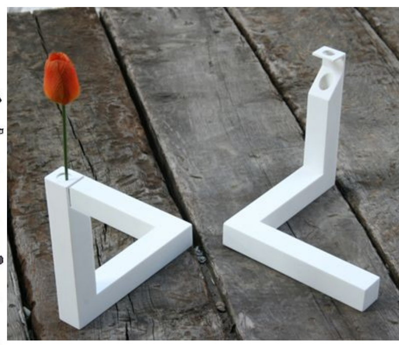


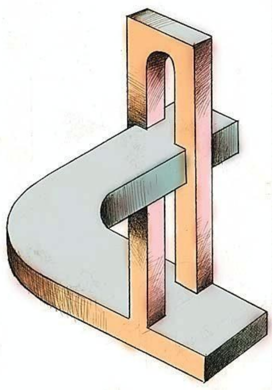
$$Life = \int_{birth}^{death} \frac{happiness}{time}\, \Delta\, time$$

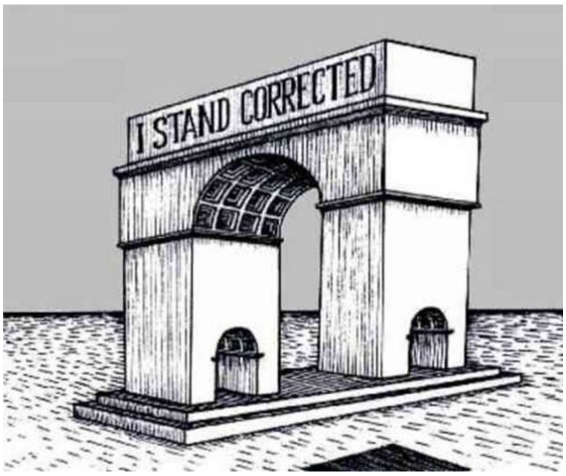
© 2011 Rebecca Dias

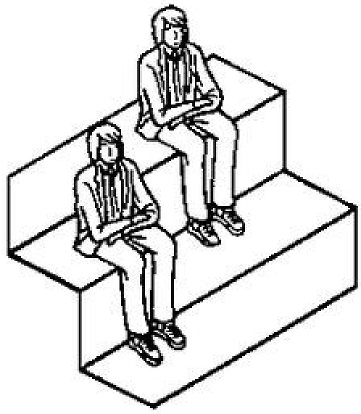
$$\int aspiri\ dn =$$

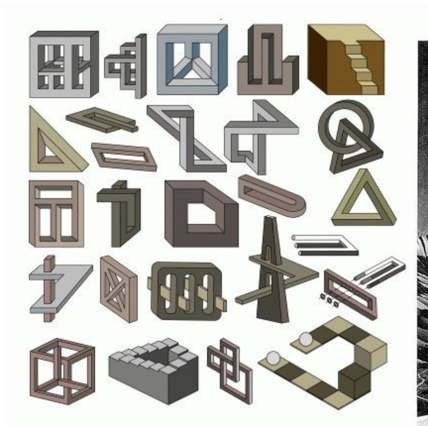


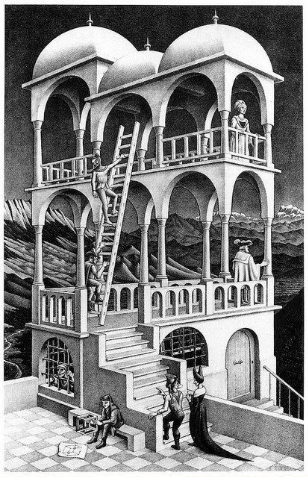


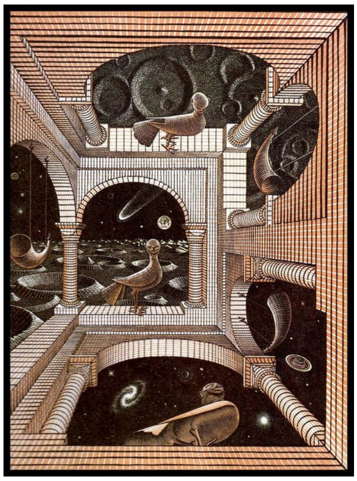


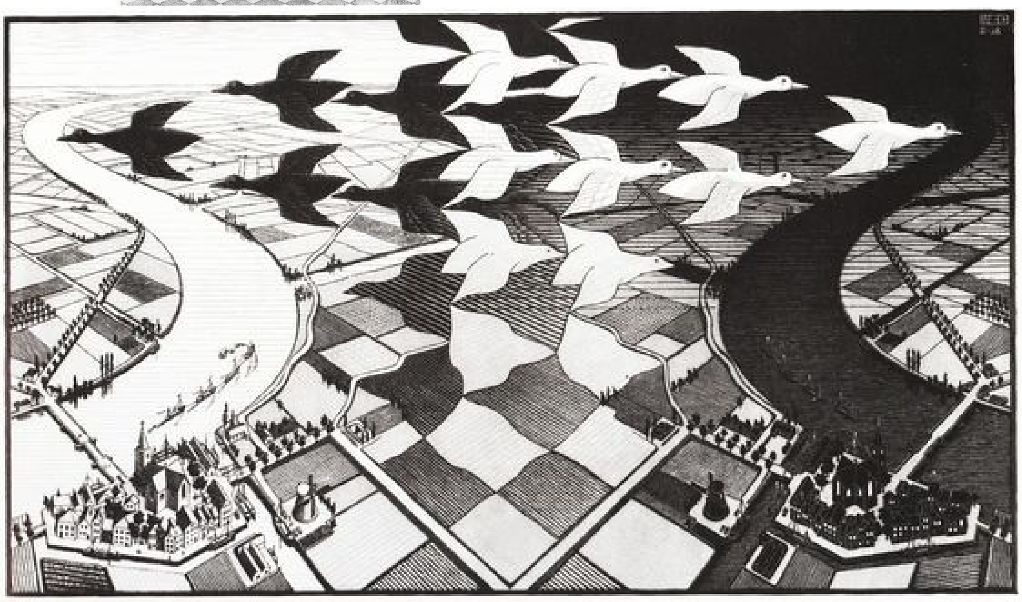

I STAND CORRECTED

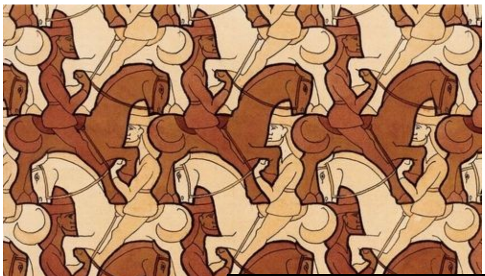


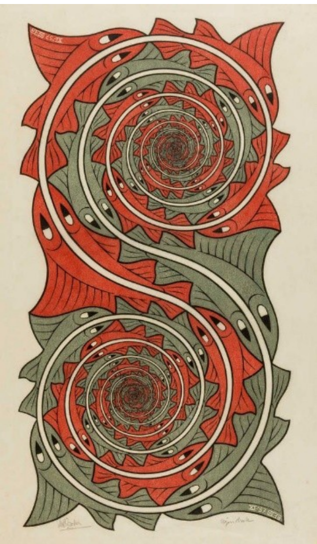


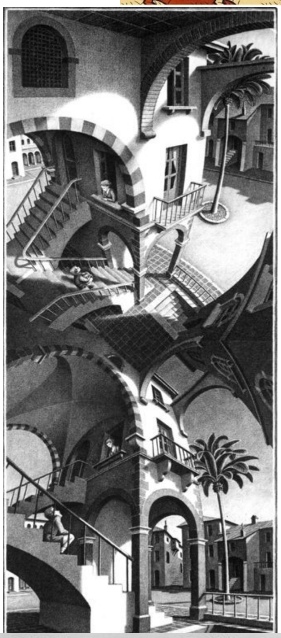


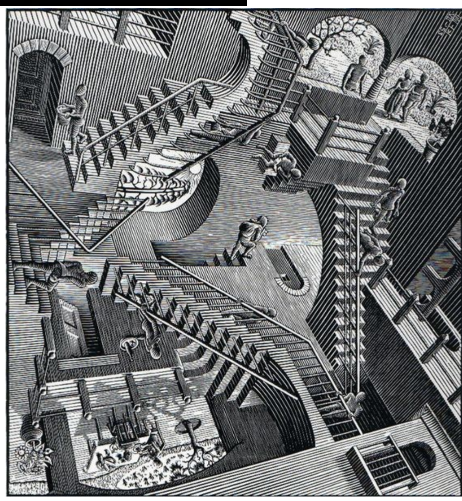












<u>Σπουδαίοι</u> σύνδεσμοι με τεράστιες συλλογές:

1. <u>**Αποδείξεις χωρίς λόγια από Geogebra (κλίκ εδώ!)**</u>
2. <u>**Επιδείξεις από Wolfram**</u>

(χρειάζεται εγκατάσταση wolfarm player σε επιτραπέζιο Η/Υ μεγέθους 1,9GB)

(απίστευτα τεράστια συλλογή από όλα τα μαθηματικά και όχι μόνο)

3. <u>**Οπτικές ψευδαισθήσεις μαθηματικές και άλλες**</u>
4. <u>**Μαθηματικές γελοιογραφίες**</u>
5. <u>**54 αποδείξεις για το Π.Θ. (στατικές)**</u>
6. <u>**1 δις προσομοιώσεις από Phet**</u>
7. <u>**Μαθηματικές προσομοιώσεις Phet**</u>

(είναι ακόμα λίγες, από τις χιλιάδες που είχαν παλιά , διότι δεν παίζει πλέον η java στους περιηγητές και τα μετατρέπουν σιγά – σιγά, σε html5 την γλώσσα που «παίζουν» όλοι οι περιηγητές, χωρίς κανένα πρόσθετο για εγκατάσταση)

8. <u>**Γεωμετρία της προοπτικής**</u>
9. <u>**Συλλογή δωρεάν βιβλίων περί τα μαθηματικά**</u>
10. <u>**Βιβλία από το mathematica.gr**</u>
11. <u>**Η τεράστια –απίστευτη-εργασία-συλλογή του Μαθηματικού κ. Τάκη Χρονόπουλου**</u>
12. <u>**Η Συλλογή του Συμβούλου Εκπαίδευσης Μαθηματικών κ. Γιάννη Καραγιάννη**</u>
13. <u>**Πλατάρος Ιωάννης, συλλογή «Μαθηματικά βίντεάκια»**</u>
14. <u>**Τα προηγούμενα και κάποια επί πλέον ως «Εκπαίδευση»**</u>
15. <u>**Και κάποια επί πλέον με μαθήματα για το sketchpad**</u>

16. **Μαθηματικά από το Tiktok**

Δοκιμάζουμε tags : algebra, geometry , trigonometry

(Υπάρχει μαθηματικό και κοινωνιολογικό ενδιαφέρον)

17. **Η πλατφόρμα brilliant.org**

Σειρές μαθημάτων μαθηματικών, από νηπιαγωγείο έως και Πανεπιστήμιο. Η εικόνες είναι πρωτεύον εργαλείο εκμάθησης . Υπάρχει δυνατότητα δωρεάν εγγραφής.

18. **Η πλατφόρμα wolframalpha που λύνει όλα τα προβλήματα χωρίς εισαγωγή του προβλήματος με προγραμματιστικό κώδικα.**

Αφορά τα πάντα, όλες τις επιστήμες, και τα μαθηματικά, από μαθηματικά ανέκδοτα μέχρι αλγεβρική τοπολογία. Υπάρχουν πολλά παραδείγματα για φιλική εισαγωγή δεδομένων.

Επίσης για επίλυση ασκήσεων σε βήματα

19. **Πλατφόρμα επίλυσης ασκήσεων https://www.symbolab.com/**
20. **Και άλλη πλατφόρμα: https://www.cymath.com/**
21. **Άλλοι λύτες προβλημάτων με βήματα: https://www.mathway.com https://quickmath.com/**

https://webmath.com/ https://math.microsoft.com/en https://www.mathpapa.com/

22. **Βίντεο, λίαν ενδιαφέρον, για το που πάνε τα πράγματα: Τα Μαθηματικά της Τεχνητής Νοημοσύνης | Κωνσταντίνος Δασκαλάκης - Πέτρος Δελλαπόρτας**

Άλλη δικτυογραφία

https://www.bulbapp.com/u/προοπτική-μέρος-β/

http://old-eclass.uop.gr/modules/document/file.php/CIVIL130/%CE%A0%CE%A1%CE%9F%CE%9F%CE%A0%CE%A4%CE%99%CE%9A%CE%9F.pdf

https://slideplayer.com/slide/4313130/

https://www.researchgate.net/publication/259604916_PLAYING_WITH_MOEBIUS_STRIPS

https://en.wikipedia.org/wiki/M%C3%B6bius_strip

https://en.wikipedia.org/wiki/Topology

https://www.youtube.com/@a-maths8031

https://www.youtube.com/@mathwindow

https://www.youtube.com/playlist?list=PLKLt4UEdBs1nFQ0jSgN5EK6OAGSym1YQ0

https://www.youtube.com/watch?v=OmJ-4B-mS-Y

https://www.youtube.com/@domainofscience/about

https://www.youtube.com/watch?v=S8uheIt9Iec

https://www.youtube.com/watch?v=r0_mi8ngNnM

https://hackernoon.com/some-types-of-geometrical-illusions

https://en.wikipedia.org/wiki/Geometrical-optical_illusions

https://en.wikipedia.org/wiki/Geometrical-optical_illusions

https://blog.numbernagar.com/2019/08/03/the-curious-case-of-the-mathematics-behind-the-illusions/

https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=&ved=2ahUKEwiB-Ku__P__AhWeUqQEHXnBDvIQFnoECCcQAQ&url=https%3A%2F%2Fweb.ece.ucsb.edu%2F~parhami%2Fpres_folder%2Ff38-math-illusions.ppt&usg=AOvVaw2MqygI8ogxEqAhnAXMkGQO&opi=89978449

https://www.math.tamu.edu/outreach/mam/illusions/

https://www.etsy.com/market/math_illusion

https://www.shutterstock.com/search/mathematics-illusion?consentChanged=true

https://www.theguardian.com/science/2020/oct/05/did-you-solve-it-the-art-of-illusion

https://www.shutterstock.com/search/geometric-illusion

https://www.slideshare.net/plataros/ss-7062612?from_action=save

https://www.slideshare.net/plataros/2-12904032?from_search=42

https://www.slideshare.net/plataros/m-1-12607576?from_search=84

https://pixabay.com/el/images/search/fractals/

https://gr.pinterest.com/litsakaminiote/%CF%86%CF%81%CE%AC%CE%BA%CF%84%CE%B1%CE%BB/

https://www.etsy.com/market/fractal_collection

https://www.vectorstock.com/royalty-free-vectors/geometric-optical-illusion-shapes-vectors

https://www.vectorstock.com/royalty-free-vectors/optical-illusions-geometrical-vectors

https://www.vectorstock.com/royalty-free-vectors/geometrical-pattern-vectors

https://imgv2-1-f.scribdassets.com/img/document/301577681/original/ceccdbad25/1579250518?v=1

https://mathoverflow.net/questions/8846/proofs-without-words

http://ignited.in/I/a/262795

https://www.maa.org/press/periodicals/convergence/proofs-without-words-and-beyond-proofs-without-words-20

https://www.qedcat.com/misc/207.html

https://mathematicsart.com/proof-without-words/

https://i.redd.it/x2uou7cwkvy51.jpg

https://paymentproof2020.blogspot.com/1980/03/proof-without-words.html

https://mathoverflow.net/questions/8846/proofs-without-words/258356#258356

https://www.youtube.com/watch?v=-Ms6fx_MlEM

hthttps://www.kidzsearch.com/kidztube/watch.php?vid=373f8d511tps://www.youtube.com/watch?v=_8qmmiz1JUI

https://www.kidzsearch.com/kidztube/watch.php?vid=373f8d511

https://www.youtube.com/watch?v=o2mbN452ywA

https://commons.wikimedia.org/wiki/File:Pizza_proof_without_words.svg

https://alchetron.com/Proof-without-words

https://www.anyrgb.com/en-clipart-hsirk

https://www.notesofdabbler.com/posts/post_2020_04_26/

https://youtu.be/whYqhpc6S6g

https://www.google.com/search?client=firefox-b-d&q=proofs+without+words+https%3A%2F%2Fwww.cut-the-knot.org

https://www.geogebra.org/m/j6H6JyQz

https://www.maa.org/press/periodicals/convergence/proofs-without-words-and-beyond